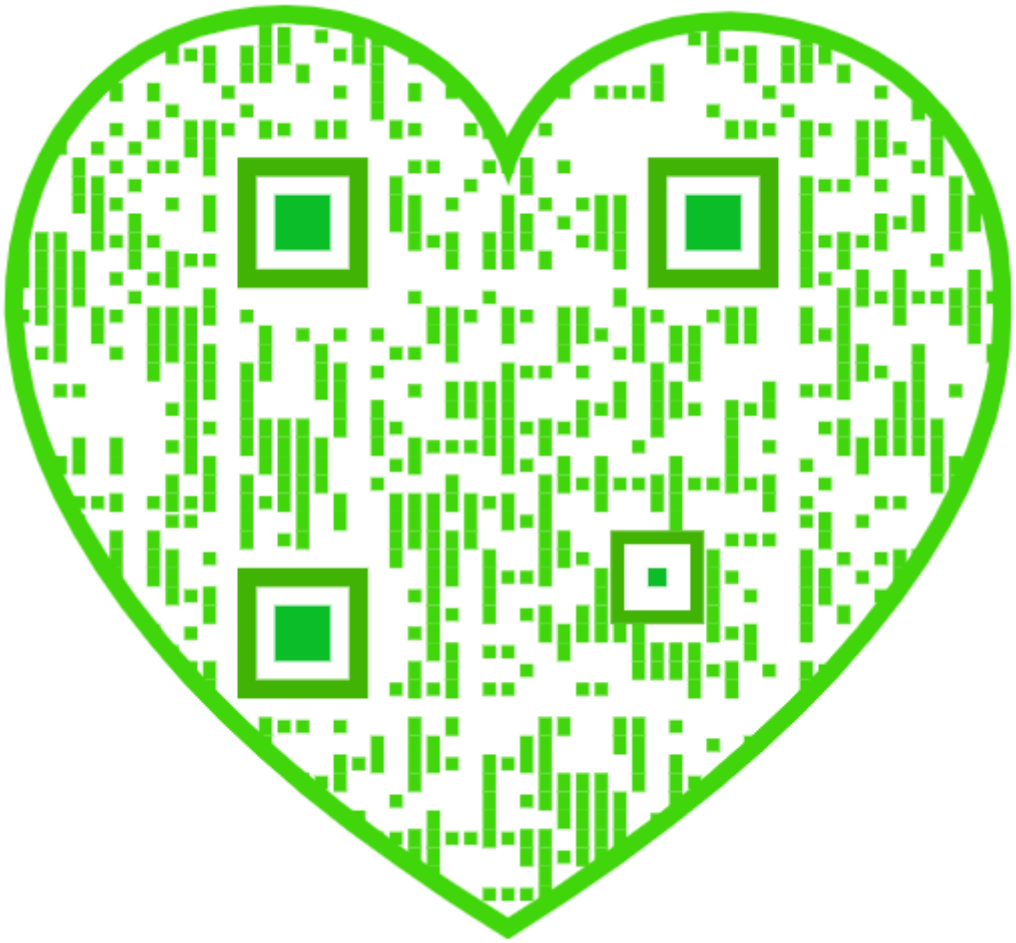

ADVANCED ARITHMETIC

BY

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"This law of organic progress is the law of all progress. Whether it be in the development of the earth, in the development of life upon its surface, in the development of society, of government, of manufactures, of commerce, of language, literature, science, art, this same evolution of the simple into the complex, through a process of continuous differentiation, holds throughout."

— HERBERT SPENCER.

BOSTON, U.S.A.

GINN & COMPANY, PUBLISHERS

The Athenæum Press

1899

E.Q.I.T 118.99.780

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PREFACE.

THE purpose of this book is to aid the teacher in making conditions favorable for the contact of the learner with mathematical realities. Since the simplest as well as the most complex relations become known only through mental activity in comparing, attention is given to an environment which shall continually induce this activity. The comparison of magnitudes includes all the other operations of arithmetic. This is apparent when we reflect that all advance in knowing is by progressive acts of analysis and synthesis. Whether we compare by means of the object or by means of its symbol, the mind should still be free to move from the vague to the definite through its own acts.

In the first book of this series it was said: "That quantity is a ratio between terms which are themselves relative is a truth which has often been pointed out, but which the work of the schools shows to be felt by few." In the minds of many bred upon the language of mathematics, the mode of expression has acquired an independence which excludes the reality. The presentation of relative magnitude as a subject of study in the elementary school has, however, met with gratifying response. Some authors, indeed, whose presentations are entirely foreign to the development of ideas of relative magnitude, have recently written urging

relations of magnitude as the objects of study; others have given the *word* "ratio" a prominent place in new editions. "A religious vocabulary without religious experiences" is of little value; a mathematical vocabulary without mathematical experiences is of no more value. The student cannot advance in any science if his attention is absorbed in the language. Non-mathematical work cannot generate mathematical ideas. Definitions of mathematics and of quantity avail little without the feeling and the intimate understanding which cause us to shape our work in accord with the illuminating idea.

The mode of dealing with the greatest common measure, percentage, longitude and time, square root, mensuration is in accord with underlying mathematical ideas. The pupil advances in the indirect establishment of relations of magnitude by living in practical contact with mathematical realities.[1]

Each subject has its own elementary ideas. The study of relative magnitude does not give a basis for inferences in history or biology, nor does attention to history give a basis for mathematics. The power to say, "The act is brave," or, "The building is fitly proportioned," begins in comparing through the senses, but the observing which leads to one of these judgments does not furnish a basis for the other. But all elementary work should be formative of a mental habit which may be fitly carried into all lines of effort.

[1] "One well-known principle underlying the acquisition of knowledge is that an idea cannot be fully grasped by the youthful mind unless it is presented under a concrete form. Whenever possible an abstract idea must be embodied in some visible representation. . . . Should it appear to any one that we thus detract from the generality of algebraic quantities, it is sufficient to reply that the system is the same which mathematicians use to assist their conceptions of advanced algebra, and without which they would never have been able to grasp the complicated relations of imaginary quantities."—SIMON NEWCOMB.

Things must be observed in various aspects if they are to be known, but to say that too much attention to mathematics or to particular things weakens is merely to make a specific application of a universal truth.[1] The presentation in this series is from the standpoint that any success is dangerous which lessens the susceptibility of the mind. The constant purpose is to promote growing power to act in new circumstances.

W. W. SPEER.

[1] "The intensity with which any form of exercise is carried on during the growing period leaves its trace, and the absence of it at the proper time is for the most part irremediable. We should hardly expect much appreciation of color in a person brought up in the dark, however good his natural endowments in this direction. Thus any lack of early experience may leave a spot permanently undeveloped in the central system."— PROF. H. H. DONALDSON.

Nor can we expect appreciation of mathematical relations if there is no opportunity for attending to them. If we leave the sense of proportion undeveloped, and leave the pupil unaware of the realities of mathematics in the elementary work, we can scarcely expect interest in relative magnitude at a later period. — AUTHOR.

INTRODUCTION.

Interdependence of the Powers. — At all stages education involves the whole personality. Continued impressibility is a condition of growing power to think and act. Attempts to substitute the analytic-synthetic acts of the adult for those of the child separate the child from realities of which he would easily possess himself if not interfered with. Through many facts we rise to laws which embrace them all and lead to other truths. So the higher mental life embraces the lower and further develops, not by disuse of the powers, but by increasingly effective use of them. Varied *self*-activity is the great characteristic which distinguishes higher from lower organisms.

Means and End. — Intelligent effort is the adaptation of means to end. Mathematics is a means of educating, just as botany and history are. From the mathematical side the purpose of the study is the discovery of the relative magnitude of things. What, then, should be the course of procedure? Manifestly the learner cannot respond to that by which he is not affected. Is there any way by which he can become affected by relations of magnitude except by his own activity [1] of sense and mind in regard to such relations?

[1] "Thus even when the relation is directly presented, *e.g.*, in the spatial relation of two simultaneously perceived objects, it is evident that attention must direct itself to this relation, and selectively bring it into mental prominence." — JAMES SULLY.

If the self-activity which gives imperfectly quantitative ideas is checked, sensitiveness to mathematical relations is not developed, and the manipulative operations the student is called on to perform in the name of mathematics become a barrier to the perception of mathematical relations. This activity *is* checked when appliances are so pressed upon the attention that relative magnitude is prevented from rising into the foreground of consciousness. It is checked whenever methods do away with the need of forming personal judgments.

It is generally admitted that we cannot know a thing in isolation. If we say, "The day is warm," "The ratio of this line to that is 3," "The color of this leaf is darker than that," or, "This is a dog," we have expressed the likeness or unlikeness of things. It is also recognized that to establish a relation between two things the things must be present in consciousness. The mode by which the mind enters into the various aspects of truth is fundamentally the same; it is by thinking. And what is thinking? We talk freely of training children to think, but what do we mean? How do we set about it? Is thinking something separate and apart, something carried on independently of sensing and feeling? Or are the things we sense and feel the things we relate? Can relations be discovered without the formation of judgments? Can a relation be discovered unless there are at least two terms? Why not? Can we discriminate, classify, form judgments, see likenesses and differences unless we compare?

From Plato down, relating has been held to be the essence of the intellectual life. Why should we shun this intellectual act, or fail to foster it in mathematics more than in any other subject?

The Mental Whole. — Whatever the mind occupies itself with becomes a whole, the object of thought for the time

being.[1] To think of the size of a house means to think of the relation of magnitude which it as a whole bears to other wholes. To think of relative times, weights, values, etc., means to think of each of the wholes in the particular relation specified.[2] Any presentation which interferes with the mental grasp of the terms of comparison as units interferes with the perception of relative magnitude.

The Mathematical Element; Coexistence; Coextension.— That certain conditions foster and others interfere with growth into mathematical ideas may become more clear when we consider what is involved in the perception of a quantitative relation.

The simplest comparison implies coexistent wholes. The object is to discover the relative magnitude of the wholes under consideration. All intermediate processes, all the separating and combining, have this end in view. It is clear, then, that each unit must be grasped *as a whole.* Now the use of the discontinuous unit directs attention not to one unit but to several. When a child's attention is centered upon the parts, as when he begins with counting or when he applies an artificial measure before making any mental note of a whole, each separately apprehended part becomes a whole; consequently, the units which should be compared are not imaged and not compared. The expression of their relation is then a formal act, not an expression of the perception of their relative magnitude. A quantity becomes discrete for an individual only when he makes it so by his own intellectual acts, his own analyzing.[3] A synthesis not accompanied by an analysis is artificial. If

[1] See "Teachers' Book," p. 17.

[2] Whether a relation is to be expressed in the language of percentage or in some other manner, the pupil should still be given opportunity to perceive the units he is to compare as coexisting and coextensive.

[3] See "Teachers' Book," p. 18.

we wish the terms $\frac{a}{b}$ to be felt, is it not a mistake to present them broken into parts, or discontinuous? By so doing we prevent the mind from grasping the terms which should be compared, and therefore from grasping their relation.

If we ask for the relative magnitude of a and b, we tacitly assert the coexistence of the magnitudes. Why, then, should we not present them as coexistent? In Mr. Spencer's words, "Coexistence implies two somethings that coexist. Two somethings cannot occupy the same space, . . . hence things cannot be known as coexistent without also being known as out of each other — at some distance from each other." What does this suggest? The presentation should foster the perception of the coextension of coexistent magnitudes of like kind. Without such perceptions there is no quantitative reasoning.[1] Upon these perceptions the constructions of mathematics rest.

Rhythm; Counting.— Consciousness consists of successive states. Rhythm is a factor in life. But these truisms have no more to do with relative magnitude as the object of mathematical study than with the ideas which characterize chemistry or botany. That the relation of coexistence is not directly grasped in the first stages of development, but is

[1] "Thus, abstract as they have now become, the units of calculation, applied to whatever species of magnitudes, do really stand for equal units of linear extension; and the idea of coextension underlies every process of mathematical analysis. Similarly with coexistence. Numerical * symbols are, it is true, purely representative; and hence may be regarded as having nothing but a fictitious existence. . . . The calculus, in general, is a means of dealing with magnitudes that do not coexist, or are not homogeneous, or both, by first substituting for them magnitudes that do coexist and are homogeneous, and afterwards re-translating these into their original forms."— HERBERT SPENCER.

* Numbers. See "Principles of Psychology," vol. ii, pp. 38, 39.

arrived at by experiences in relating, is an additional reason for conditions which elicit activity in comparing. We have chemical, astronomical, and biologic rhythms; but we do not, therefore, begin these subjects with exercises in counting. Whether comparing things in respect to color, beauty, taste, or magnitude, the mind moves from the whole it grasps to the parts. If we wish the child to see relations of likeness and unlikeness between an orange and an apple, we bring the orange and the apple before him. We do not put parts before him and ask him to look at them. Why not pursue a similar course if we wish him to see the likeness or difference in the magnitude of things?

If we believe the appreciation of a picture or a poem comes through absorbing the mind in details before the whole has been seen, if we believe the child likes to act upon elements which the adult has dissociated from the concrete whole, then, naturally we make counting the basis in mathematics. But if we believe that in all thinking, whatever the mind attends to becomes the whole, that it can relate only what it grasps, we naturally present coexistent, coextensive magnitudes. Analyzing for himself, the learner does not lose his grasp of the things to be compared. He advances by progressive analysis and synthesis according to his capacity.

Self-Activity the Means of Advance. — Progress in any science implies growth. Growth depends upon self-activity. Those who accept these ideas cannot consistently substitute analyses, rules, or mechanical measuring for the self-activity through which the mind grows into mathematical ideas. I say "grows" because only through this evolutionary process can there arise an inner relation answering to an outer one; only thus can objective truth be realized. Growth into mathematical, as into other ideas, is the slow, gradual result of continuous judging, and an environment

which continually excites such activity is the means of teaching mathematics. This activity in judging of relative magnitude no more involves mental strain than the activity expressed in the judgment, "The book is green," "This candy is sweeter than that," or "The boy is good." All these ideas are the result of individual activity in comparing.

Digressing slightly, I would call the attention of teachers to the fact that expressions, even though they are the genuine products of mental action on the part of the child, may not signify to him what they do to the adult. They can do so only through many and varied experiences.[1] "The boy is good," "The ratio of 8 to 4 is 2," are ideas which become more complete and definite with developing experiences.[2]

Through Indefinite to Definite. — Mathematics offers no exception to the rule that the mind advances from the indefinite to the definite by its own acts. The mental process by which the child becomes aware of marked differences in color or magnitude is the only process by which he can become aware of finer distinctions and closer likenesses. Realization of this forbids attempts to create definite ideas by a use of mechanical apparatus which subordinates mental action.

In the first book of this series attention was called to the essential likeness of reasoning in mathematics and in other subjects. In the words of Mr. John Fiske: "Between the two kinds of reasoning the only essential difference is the degree of definiteness to which they are respectively developed." And Mr. Herbert Spencer has pointed out that[3]

[1] See "Teachers' Book," p. 15.

[2] A child of four, reading, "Tom has a knife. It is a good knife," said with surprise: "What makes them say that? Knives can't be naughty, can they?" This suggests the varied experiences needed to give meaning to abstract and general terms.

[3] "A man who has walked a mile in fifteen minutes, and, observing that he has a quarter of a mile still to go, infers the time it will take

"Every act of quantitative reasoning is qualitative in its initial stages." That is, the mind grasps things vaguely as wholes and relates them as more and less before it arrives at their exact relation by the aid of artifices of calculation. The mental act may be so rapid that ordinarily we fail to note it, but progress is through the free movement of the self-active mind. The power to establish definite relations is but a higher development of the power to establish indefinite relations.

Environment. — The environment of the school includes the material, the demands, methods; in short, all to which the child conforms himself.

As implied above, growth into mathematical ideas is not the result of using definite measures or definite language, but of continued contact with *relative* magnitude. There is no such contact unless the mind brings things into quantitative comparison. It cannot do so if energy is engrossed in something else, — if a growing sensitiveness to mathematical realities is prevented by formal work, by absorption in appliances for securing outside products. As the school aids the evolution of ideas in botany or history by conditions which promote growing consciousness of these phases of truth, so it should do in mathematics. Allowing for natural aptitudes and capacity, the child attends to that in which it sees others interested. The impressions we receive are the cause of our reactions. If, then, the school places a fictitious value upon outside effects and limits sensory and motor reaction to the stimuli of mechanical operations, the experiences which cause the progressive development of the mind do not take place. By mistaking the means for the end

to reach his destination, does not primarily infer *three minutes and three-quarters*, he primarily infers a *short time* — a time indefinitely conceived as less than ten minutes and more than one." — HERBERT SPENCER.

we too often shut the child out from these experiences. As freedom may be lost by exalting special forms of free government, so mathematics may be obscured by its appliances.[1] The measuring which develops mathematical ideas is not a series of mechanical operations akin to the automatic practice of the carpenter with his rule, but the mental comparing, without which things, as we know them, do not exist for us. The so-called objective method may no more ignore the perceiving mind than the mind may ignore the continual verification of experiment.

The learner may make due use of a measure in testing a judgment after he has made the judgment. To supplement the activity of mind and senses is one thing, to supersede it is another. We check development and rob the pupil of the experiences which expand the idea and the mind, when we limit energy to mechanical apparatus or to processes for securing particular results.

"The study of mathematics," said Professor Sylvester, "is unceasingly calling forth the faculties of observation and comparison." By examples seemingly far removed from sense he showed that mathematical discovery is always the outgrowth of sensory and intellectual activity. Reliance upon artificial measuring, or that vicarious analyzing which presents the terms already divided, is but a variation of the old plan of getting results by rule. Why should a child think when a rule will serve? Why should he compare the magnitude of things, why judge of their relative magnitude, when a foot rule, an inch, or a pound weight will do away with the need of mental movement? Why should he look at plants when the catalogue of the naturalist is ready to

[1] "All the devices familiar to the reader of Euclid . . . are simply devices for bringing a given pair of space relations directly into consciousness, so that their equality or inequality may be recognized by direct inspection."— JOHN FISKE.

his hand? Perhaps for no reason except that he cannot advance in any science, or in the power of definite thinking, without continued mental activity.

The school should economize energy by a presentation which attracts attention to the essential. It should reënforce self-activity by an environment fitted for the developing powers.

Formal Analyses. — Another phase of counterfeit activity in mathematics is formal, logical analysis. As making and measuring by rule give artificial products, so do analyses which make children the "parrots of other men's thinking." They stifle thought and conceal the condition of the learner. Professor Jevons invented a logical machine, but he did not claim that it acted intelligently. Manipulating formal analyses may satisfy the demands of the school, but not of life.

The repetition of the adult's "sinces" and "therefores" is not reasoning,[1] nor is mental power increased by requiring the student continually to set forth every step by which a conclusion is reached. The work should call for personal expression, and correct decision is evidence that the mind is not moving amiss. Why explain again and again the reasons for the inference that in 6 hours we may walk $2x$ miles, if in 3 hours we walk x miles, any more than for the inference that the apple is good? The child is not called upon to analyze the acts by which he classifies this as an orange, that as a pear. It is sufficient that he decides cor-

[1] "The doctrine that we can discover facts, detect the hidden processes of nature, by an artful manipulation of language, is so contrary to common sense that a person must have made some advances in philosophy to believe it."— JOHN STUART MILL.

"High Air-castles are cunningly built of Words, the Words well bedded also in good Logic-mortar; wherein, however, no Knowledge will come to lodge."—THOMAS CARLYLE.

rectly. If bodily and intellectual acts did not sink into the automatic, progress would soon be at a standstill.

By a multitude of experiences the child is able so swiftly to compare two pails, and to infer that the larger will hold a larger amount, and to act accordingly, that perception, inference, and choice seem one; and who would not think it absurd to require verbal proof of the wisdom of the choice? Mental as well as physical experiences become organically registered, and the mind does not dwell upon familiar perceptions any more than it does upon the series of acts involved in reading or playing the piano. The mind must analyze in order to read a page, but what would be the effect of dwelling upon each separate word, syllable, and letter?

There is no more reason for asking, "How do you see this?" in mathematics than in any other subject. The result of attempts to apply the introspective method to adult consciousness does not recommend this plan with children. The tendency to suspend judgment and to rest opinions upon facts instead of upon conjecture is not the result of dwelling upon details. The demand for continually particularizing every step by which we reach a conclusion interferes with the forward reaching functions of the mind and with the growing power to separate the essential from the non-essential. It is the continued adjustment of the mind to things which makes their connections familiar and necessary.

Motor Activity. — As previously urged, methods which separate the thinking being from the being who senses, feels, and acts are mischievous. At no stage can we afford to neglect the motor activity which promotes free circulation and supplies the brain with the blood necessary for attention and achievement. Moreover, the dependence of developing intelligence upon developing nervous structure urges

provision for sensory-motor processes, which supply nerve energy and excite intellectual responsiveness in given directions.

There is no reason for continuing to neglect the plain teaching of science concerning the bearing of varied movement upon mental nutrition. Sensing leads not only to imagination and inference but to outward expression, and the school should open the way for the inner life to perfect itself in action.

The bearing of the tactual and muscular senses upon development of motor intuitions and upon the blending of motor intuitions into ideas of distance, size, etc., are too familiar to need repeating. Adapted movement is an important factor in forming and fixing mathematical judgments, but it must not be forgotten that there may be activity of the hand as well as of the tongue without reflecting the mind, — activity which represses instead of develops. Motor action is a means of inducing and correcting needful judgments as well as an expression of what is seen, but motor action in drawing, cutting, making, etc., should be the genuine expression of action within the doer. Making by rule and measure no more produces a mental structure capable of picturing conditions, or gives a growing sense of mathematical relations, than does repeating formulas without the experiences which give them significance.[1] Right manual training means mental training, and it is an essential factor of education. We do not meet the needs of the child by giving him a rule or a tool for producing a particular result. That which is with-

[1] "The intellectual factor of manual labor is never very large even in the first construction of a new type of product. The moral education in manual training in the way of perseverance, patience and plodding industry is a far greater educational factor than the intellectual factor." — W. T. HARRIS, "Report Com. on Pedagogy," 1889.

out and that which is within is brought into harmony by action which is the outgrowth of perception, imagery, and will. Effects transcending the receptive, analytic-synthetic power of the mind are fatal to self-activity and to intellectual and moral growth.

A child may apply a foot rule three times to the length of a box, and twice to its width, and then mark off the same number of spaces on another piece of wood or pasteboard, and, by a series of similar processes, construct articles innumerable with scarcely a glimmering of proportion or a stirring of the representative power. Year after year the carpenter measures without more progress in mathematics than the farmer, just as year after year we may handle the crucibles of the chemist without advancing in chemistry, or repeat the formulas of the thinkers without thinking.

The equations of mathematics express real, not formal, truths; and the only means of contact with these truths is through the activity of the mind in regard to the relative magnitude of things. There is no other mode of adjustment. Newton urged that if ever the action of observant senses and mind in regard to the mathematical relation of things was displaced by artifices of calculation, then would the study of mathematics degenerate into something little better than handicraft dexterity.

Whenever the method of the school fails to preserve the balance between impressibility and outer activity, whenever it centers upon outward products, whether secured by formulas or measures, to the neglect of mental receptivity, responsiveness, and growing power, it is non-developing.

Concrete and Abstract. — Continued activity in a varied environment ensures progressive abstraction. Gradually the mind is freed from the concrete and the particular.[1] The power to think in general and abstract terms is the product

[1] See "Teachers' Book," pp. 19, 20.

of many experiences through which the conscious becomes unconscious, and memories become organic. As from particular plants and minerals we rise to a comprehension of the types with which science deals, so we advance in mathematics.

No definite lines can be drawn between the power of perceiving and the power of inferring relations, nor between the power to compare by means of visible things and the power to compare relations without the aid of the things displaying them. If we attribute to the childish mind the sense of proportion, of ratios, which belongs to the adult, and so neglect the manifold experiences,[1] the acts of associating and dissociating by which the pupil grows into definite ideas, we fail to teach mathematics. Every fresh experience, every new perception of the concrete, gives greater power to think in general and abstract terms.

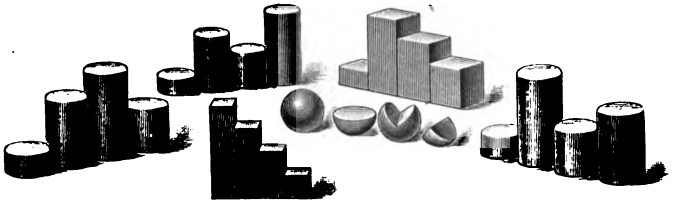
Not merely arithmetic, but the science of mathematics is a whole, and in the elementary work a bias is given to the mind which fits or unfits it for discovery. Not by dwelling in turn upon the language and devices of arithmetic, geometry, and algebra, but by a presentation that promotes those unifying processes which take place within the mind, do we promote progress in mathematics. Only by the evolution of interest, action, and power is the mind prepared for high abstraction. Stereotyping upon the brain isolated images which "live alone and die alone incapable of fecundation" is not the means by which to develop that representative power which is the mark of intellectual progress. "The higher processes of the mind in mathematics lie at the very foundation of the subject." The relations arrived

[1] "Men fail to judge correctly because they have not had that sufficient instruction by the senses which would justify making a conclusion. We must contrive extra and special means by which their first impressions shall be corrected and enlarged."—FARADAY.

at through symbols are just as real as those directly discerned through things, but symbols gradually lose vitality if those who use them separate themselves from realities. If there is an enforced separation in childhood, we cannot wonder that this separation continues in after life.

Any work which is not suited to the receiving and transforming power of the learner's mind is harmful. Growth is a matter of time and nutrition, but if we work with nature, the child will do easily what he might otherwise be unable to do at all.

ADVANCED ARITHMETIC.



d c b a

1. Tell all you can about the units a, b, c, and d.

2. What ratios do you see?

3. 2 is the ratio of which units?

4. Compare b with each of the other units. Compare a with each. Compare c with each. Compare d with each.

5. If d is 1, each of the other units equals what part of 1?

6. What is the ratio of 1 to $\frac{1}{2}$? to $\frac{1}{4}$? of $\frac{3}{4}$ to $\frac{1}{4}$? of 1 to $\frac{1}{2}$ of $\frac{1}{2}$? of 1 to $\frac{1}{8}$ of $\frac{3}{4}$?

7. What is the sum of $\frac{1}{4}$ and $\frac{1}{2}$? of $\frac{1}{2}$ and $\frac{3}{4}$?

8. Make sentences like this: 1 is the ratio of $\frac{1}{2}$ of 1 to $\frac{2}{3}$ of $\frac{3}{4}$.

9. Make sentences like this: 2 is the ratio of $\frac{1}{2}$ to $\frac{1}{3}$ of $\frac{3}{4}$.

10. If the unit 1 costs x¢, what part of x¢ does each of the others cost?

11. If the unit $\frac{1}{4}$ costs 7¢, the cost of each of the other units equals how many 7¢?

12. If c is 1, what is the name of each of the other units?

13. Why do you call a $\frac{1}{3}$? Why did you before call it $\frac{1}{4}$?

14. What is the ratio of $1\frac{1}{3}$ to $\frac{1}{3}$? to $\frac{2}{3}$?

15. What is the ratio of 1 to $\frac{1}{3}$? to $\frac{1}{2}$ of $\frac{2}{3}$?

16. What is the ratio of $\frac{2}{3}$ to $\frac{1}{3}$? to $\frac{2}{3}$? to $1\frac{1}{3}$?

17. If b is 1, what is each of the other units?

18. Compare each with the other three.

19. Why do you call d 2? a $\frac{1}{2}$?

20. c equals how many halves of b?

21. 3 is the ratio of c to what part of b?

22. What is the ratio of $1\frac{1}{2}$ to $\frac{1}{2}$?

23. When d is 4, what is a?

24. When d is 2, what is b?

25. Cut, free-hand, out of paper, the units 1, $\frac{3}{4}$, $\frac{1}{2}$, and $\frac{1}{4}$.

26. Compare these units until familiar with their ratios.

27. Compare $\frac{1}{2}$ of each unit with $\frac{1}{2}$ of each of the others. Write the comparison.

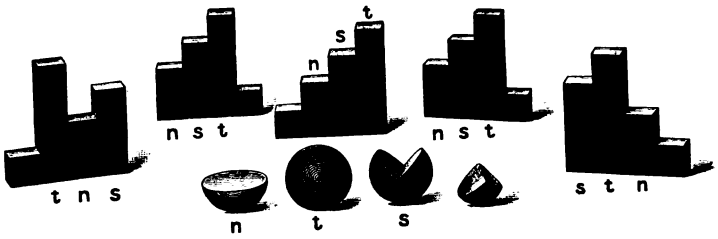
28. What is the ratio of 1 to $\frac{1}{4}$? to $\frac{1}{2}$? of $\frac{3}{4}$ to $\frac{1}{2}$?

29. Look at a, b, c, and d. What is the ratio of a to $\frac{1}{2}$ of a? of b to $\frac{1}{2}$ of b? of c to $\frac{1}{2}$ of c? of d to $\frac{1}{2}$ of d?

30. What is the ratio of $\frac{1}{4}$ to $\frac{1}{2}$ of $\frac{1}{4}$? of $\frac{1}{2}$ to $\frac{1}{8}$ of $\frac{1}{2}$?

31. What is the ratio of 100 to $\frac{1}{2}$ of 100? to $\frac{1}{8}$ of 100? to $\frac{1}{2}$ of 50?

32. The ratio of a smaller to a larger floor equals the ratio of $\frac{1}{3}$ of $\frac{1}{2}$ to $\frac{1}{2}$. 20 yd. of carpet are required to cover the smaller floor. How many yards are required to cover the larger? What is the ratio of the time required to paint the larger floor to the time required to paint the smaller?



1. In each group, if n is 1, what is the name of each of the others?

2. In each group touch the $\frac{1}{2}$, the 2, the $\frac{3}{2}$, the 1.

3. Observe a group and think the order in which the units are placed. Observe another; another.

4. Observe each group and tell the names from right to left; from left to right.

5. Practice thinking the units from right to left and from left to right in each group.

6. Try to think the order in which each group is arranged without observing the units.

7. Call s 1 and name the other units in each group. Touch the $\frac{1}{3}$, $\frac{2}{3}$, 1, $\frac{4}{3}$.

8. Practice imaging the units in each group in order.

9. Call t 1 and name the units in each group.

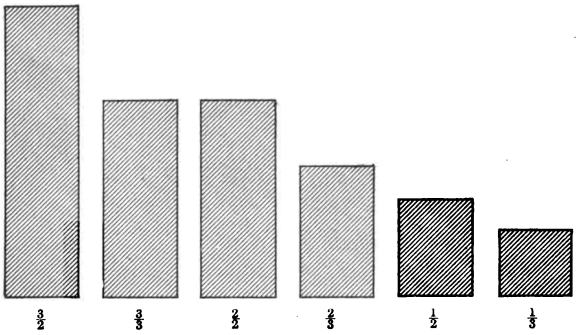
10. Practice imaging the units in order.

11. Take a set of solids which you can call $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1. Arrange in a group. Tell the names from right to left, or left to right, without observing. Arrange again and think or tell names.

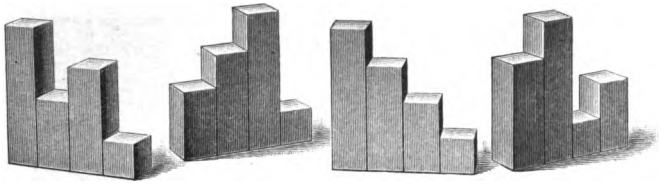
12. How many different arrangements of the units in a group of four solids can you make?

13. If $\frac{1}{4}$ of a bunch of firecrackers is worth $2¢$, what is $\frac{3}{4}$ of a bunch worth?

14. $\frac{3}{4}$ of a pound of candy is worth $30¢$. What is $\frac{1}{2}$ of a pound worth?



1. What is the ratio of $\frac{3}{2}$ to $\frac{3}{3}$?
2. What is the ratio of $\frac{2}{2}$ to $\frac{2}{3}$?
3. What is the ratio of $\frac{1}{2}$ to $\frac{1}{3}$?
4. What is the ratio of $\frac{2}{3}$ to $\frac{2}{2}$?
5. What is the ratio of $\frac{1}{3}$ to $\frac{1}{2}$?
6. 2 is the relation of what to $\frac{1}{2}$? $\frac{3}{2}$ is the relation of what to $\frac{1}{2}$?
7. $\frac{2}{3}$ is the relation of what to $\frac{3}{4}$? $\frac{4}{3}$ is the relation of what to $\frac{3}{4}$?
8. 30¢ is the cost of $\frac{1}{2}$ lb. of tea. 2·30¢ is the cost of how much tea?
9. What is the sum of $\$\frac{1}{2}$ and $\$\frac{1}{4}$? of $\frac{1}{2}$ lb. and $\frac{1}{4}$ lb.? of $\frac{1}{4}$ oz. and $\frac{1}{2}$ oz.?
10. $\frac{1}{2}$ and what equal $\frac{3}{4}$?
11. $\frac{1}{2}$ and what equal $\frac{5}{4}$? $1\frac{1}{2}$?
12. $\frac{1}{4}$ and what equal $\frac{3}{4}$?
13. The ratio of the amount of work done by A to that done by B equals the ratio of $\frac{6}{6}$ to $\frac{3}{3}$. What is the ratio of the money A earns to the money B earns? of $\frac{1}{8}$ of the money B earns to $\frac{1}{8}$ of the money A earns?
14. What is the ratio of the cost of 2 oranges to the cost of 3 oranges? of the cost of $\frac{1}{2}$ of a dozen to the cost of $\frac{3}{4}$ of a dozen?



1. What ratios do you see? Write the ratios that you see.

2. If c is 1, what is the name of each of the other units?

3. Why do you call b $\frac{2}{3}$? a $\frac{1}{3}$? d $\frac{4}{3}$?

4. If d is 1, what is the name of each of the other units?

5. Compare the 1 with each of the others. Compare the $\frac{3}{4}$ with each. Compare the $\frac{1}{2}$ with each. Compare the $\frac{1}{4}$ with each.

6. What is the ratio of $\frac{1}{2}$ to $\frac{3}{8}$ of $\frac{3}{4}$? to $\frac{1}{2}$? to $\frac{1}{4}$?

7. $\frac{1}{2}$ of $\frac{1}{2}$ equals what part of 1? $\frac{1}{2}$ of $\frac{1}{4}$ equals what part of 1?

8. Show me the blackboard. Show $\frac{1}{2}$ of it. Show $\frac{1}{4}$ of it. Show $\frac{3}{4}$ of it.

9. Compare the blackboard with its $\frac{3}{4}$, its $\frac{1}{2}$, its $\frac{1}{4}$. Look at the unit $\frac{4}{4}$.

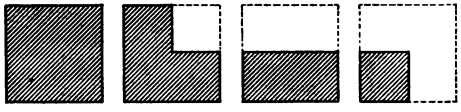
10. What is the name of the two equal units in the unit $\frac{4}{4}$? What four equal units do you see in the unit $\frac{4}{4}$? What unequal units do you see? The units $\frac{3}{4}$ and $\frac{1}{2}$ are unequal.

11. Compare corresponding parts of the units. *Ex.* $\frac{1}{2}$ of one with $\frac{1}{2}$ of another.

12. The sum of $\frac{1}{2}$ and $\frac{1}{4}$ of the cost of a horse equals what part of its cost?

13. $\frac{1}{2}$ yd. of ribbon costs 20¢. $\frac{3}{2}$ of 20¢ equal the cost of what part of 1 yd.?

14. B and A own an equal amount of land. A sells $\frac{3}{4}$ of his and B $\frac{1}{2}$, at the same price per acre. What is the ratio of the money B receives to the money A receives?



1. What ratios do you see?

2. If the largest unit is 1, what is the name of each of the others?

3. What is the ratio of 1 to each of the others? of $\frac{3}{4}$ to each? of $\frac{1}{2}$ to each? of $\frac{1}{4}$ to each?

4. What part of $\frac{1}{2}$ do you see in the $\frac{1}{4}$? What part of $\frac{3}{4}$ do you see in the $\frac{1}{4}$? in the $\frac{1}{2}$?

5. What is the sum of $\frac{1}{2}$ and $\frac{1}{4}$? of $\frac{3}{4}$ and $\frac{1}{4}$? of $\frac{3}{4}$ and $\frac{1}{2}$? of $\frac{1}{2}+\frac{1}{4}+\frac{3}{4}$? How many $\frac{1}{2}$'s in $\frac{3}{4}$?

Ans. There are $\frac{3}{2}$ of $\frac{1}{2}$ in $\frac{3}{4}$.

6. How many $\frac{3}{4}$ in 1? $\frac{1}{2}$ is how much less than $\frac{3}{4}$?

7. Compare 100 with $\frac{3}{4}$ of 100; with $\frac{1}{2}$ of 100; with 125.

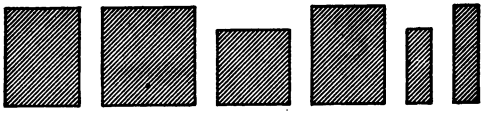
8. What is the ratio of x to $\frac{1}{3}$ of $\frac{3}{4}x$? to $\frac{x}{3}$? to $\frac{x}{4}$?

9. Make three questions similar to the following: If I paid $\$\frac{1}{2}$ for some ribbon, and $\$\frac{3}{4}$ for a book, what did they both cost?

10. Make three questions like the following: If $1\frac{1}{4}$ lb. of coffee cost $x¢$, what part of $x¢$ does 1 lb. cost?

11. One ton of coal weighs how many times as much as $\frac{1}{4}$ of a ton? It weighs how many times as much as $\frac{1}{2}$ of $\frac{1}{2}$ of a ton? It weighs how many times as much as $\frac{1}{3}$ of $\frac{3}{4}$ of a ton?

12. Four is the ratio of 1 ton of coal to what? Four is the ratio of 1 ton of coal to what part of $\frac{1}{2}$ ton? Four is the ratio of 1 ton of coal to what part of $\frac{3}{4}$?



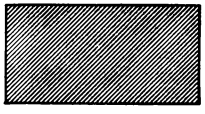
1. What is the ratio of $\frac{4}{4}$ to $\frac{4}{3}$?
2. What is the ratio of $\frac{3}{4}$ to $\frac{3}{3}$?
3. What is the ratio of $\frac{1}{4}$ to $\frac{1}{3}$?
4. What is the ratio of $\frac{3}{3}$ to $\frac{3}{4}$?
5. What is the ratio of $\frac{1}{3}$ to $\frac{1}{4}$?
6. If the ratio of $\frac{4}{4}$ to $\frac{3}{3}$ is a, what is the ratio of $2 \cdot \frac{4}{4}$ to $\frac{3}{3}$? of $2 \cdot \frac{4}{4}$ to $2 \cdot \frac{3}{3}$?
7. If the ratio of $\frac{4}{4}$ to $\frac{3}{3}$ is m, what is the ratio of $\frac{1}{2}$ of $\frac{4}{4}$ to $\frac{1}{2}$ of $\frac{3}{3}$?
8. The ratio of the cost of 1 yd. of silk to the cost of 1 yd. of muslin is 3. What is the ratio of the cost of 20 yd. of silk to the cost of 20 yd. of muslin?
9. If at the above rates a lady pays an equal amount for muslin and for silk, what is the ratio of the number of yards of muslin purchased to the number of yards of silk?
10. The ratio of the rectangle m to the rectangle b is 3. What is the ratio of the sum of the two rectangles to b? to m? What is the greatest part of m which is an exact measure of each?
11. The ratio of a rectangle, a, to the rectangle b is 1. What is the ratio of the sum of $\frac{1}{2}$ of b and $\frac{3}{4}$ of a to a? to b?
12. x and y are two equal rectangles. The greatest common measure of $\frac{1}{4}$ of x and of $\frac{3}{4}$ of y is what part of y? what part of $\frac{3}{4}$ of y?
13. A merchant cuts two pieces of lace, one 8 yd. in length, the other 6 yd., into the longest equal remnants. Each remnant equals what part of 6 yd.? what part of 8 yd.? What is the ratio of $\frac{1}{3}$ of 6 yd. to $\frac{1}{4}$ of 8 yd.?

14. What is the ratio of $\frac{1}{2}$ of 1 yd. to $\frac{3}{4}$ of 1 yd.? The greatest common measure of $\frac{1}{2}$ of 1 yd. and $\frac{1}{4}$ of 1 yd. equals what part of $\frac{1}{2}$ of 1 yd.?

15. Draw equal rectangles of different dimensions and show the ratio of $\frac{1}{4}$ of 1 to $\frac{1}{8}$ of the other.

16. The floor a equals $\frac{2}{3}$ of $\frac{3}{4}$ of the floor b. What is the ratio of the amount of work done in sweeping the two floors?

1. Draw a rectangle and separate it into eight equal parts. Show the two largest equal parts of the rectangle. Show the four largest equal parts. Show $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, and $\frac{4}{4}$ of the rectangle.



2. What is the ratio of the rectangle to each of these parts? of $\frac{1}{2}$ of it to each? of $\frac{3}{4}$ of it to each?

3. What is the ratio of $2\frac{1}{4}$ to $\frac{1}{4}$? to $\frac{3}{4}$?

4. What is the ratio of 5 to $2\frac{1}{2}$? to $1\frac{1}{4}$?

5. 3 is the ratio of what to $\frac{1}{4}$? 2 is the ratio of what to $\frac{1}{2}$?

6. 2 is the ratio of what to $\frac{3}{4}$? What is the ratio of $1\frac{1}{2}$ to $\frac{3}{4}$?

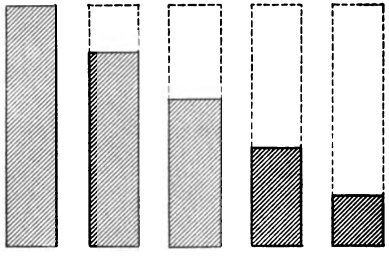
7. A farmer has $1\frac{1}{2}$ bu. of red clover seed and $\frac{3}{4}$ bu. of white. How much more of the red than the white?

8. If $1\frac{1}{2}$ bu. of corn is worth $\$x$, what part of $\$x$ is 1 bu. worth?

9. What is the ratio of 1 bu. to 1 pk.? of $\frac{1}{2}$ bu. to $\frac{1}{2}$ pk.? of $\frac{3}{4}$ bu. to $\frac{3}{4}$ pk.?

10. What is the ratio of a 2-in. cube to a cu. in.? of $\frac{1}{2}$ of a 2-in. cube to $\frac{1}{2}$ of a cu. in.? of $\frac{1}{8}$ of a 2-in. cube to $\frac{1}{8}$ of a cu. in.?

11. Draw a 2-in. square. Draw a sq. in. What is the ratio of the 2-in. square to the sq. in.? of $\frac{1}{4}$ of the 2-in. square to $\frac{1}{4}$ of the sq. in.?



1. What ratios do you see? Write the ratios that you see.

2. If c is 1, what is the name of each of the others?

3. What is the ratio of $1\frac{2}{3}$ to each of the others? of $1\frac{1}{3}$? of 1? of $\frac{2}{3}$? of $\frac{1}{3}$?

4. If b is 1, what is each of the others?

5. Learn these names: $1\frac{1}{2}$, 1, $\frac{3}{4}$, $\frac{1}{2}$, $\frac{1}{4}$.

6. Compare 1 with each of the others. Compare $\frac{1}{2}$ with each.

7. If a is 1, what is each of the others?

8. What is the ratio of $\frac{3}{5}$ to each of the others? of $\frac{4}{5}$? of 1?

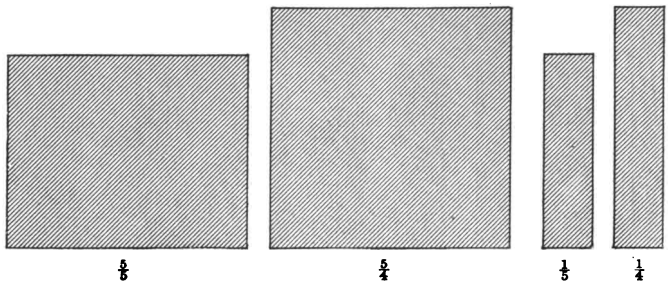
9. If the $\frac{3}{5}$ equal 45 sq. ft., what part of 45 sq. ft. does each of the others equal?

10. If the $\frac{4}{5}$ is worth $\$x$, what is each of the other units worth?

11. What is the ratio of $\frac{2}{5}$ of 15 to $\frac{4}{5}$ of 15? of $\frac{2}{5}$ of x to $\frac{4}{5}$ of x?

12. If x is 7 times as large as y, $\frac{1}{2}$ of x is how many times as large as $\frac{1}{2}$ of y? What is the ratio of $5x$ to $5y$? of $\frac{1}{4}$ of x to $\frac{1}{4}$ of y?

13. Use different sets of prisms having the ratios 1, 2, 3, 4, 5. Use blackboard drawings having the same relations.



1. What is the ratio of $\frac{5}{8}$ to $\frac{5}{4}$? of $\frac{4}{8}$ to $\frac{4}{4}$? of $\frac{1}{8}$ to $\frac{1}{4}$?
2. What is the ratio of $\frac{5}{8}$ to $\frac{5}{2}$? of $\frac{2}{8}$ to $\frac{2}{2}$? of $\frac{1}{8}$ to $\frac{1}{2}$?
3. What is the ratio of $\frac{5}{2}$ to $\frac{5}{8}$? of $\frac{2}{2}$ to $\frac{2}{8}$? of $\frac{1}{2}$ to $\frac{1}{8}$?
4. What is the ratio of $\frac{5}{3}$ to $\frac{5}{8}$? of $\frac{3}{3}$ to $\frac{3}{8}$? of $\frac{1}{3}$ to $\frac{1}{8}$?

1. There are two spoons at each plate. If one spoon is taken from each plate, what part of all the spoons is taken?

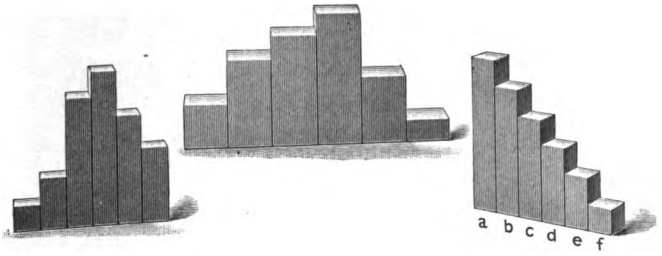
2. If you give Mary $\frac{1}{2}$ of each of the apples on a plate, what part of the apples do you give her? How do you divide the apples into two equal parts? If you give her $\frac{1}{3}$ of each apple, what part of all the apples do you give her?

3. Separate an apple into two equal parts. Divide the apple equally between 3 boys. If you give each boy $\frac{1}{3}$ of each half, what part of the apple does he receive? Each half is divided into how many parts?

4. Separate a piece of paper into two equal parts. Divide the two parts equally between 5 pupils. If you give each pupil $\frac{1}{3}$ of each part, what part of the paper does he receive?

5. Separate a piece of paper into five equal parts. If you give Harry $\frac{1}{2}$ of each 5th of the unit, what part of the unit do you give him?

6. $\frac{1}{3}$ of each $\frac{1}{7}$ of b equals what part of b?



1. What ratios do you see?

2. How often does the ratio 3 occur? the ratio 2?

3. Find the unit of which d is the largest exact measure.

4. What is the largest common measure of e, c, and a? e is the largest common measure of which units?

5. What part of e is the largest common measure of e and d?

6. What part of d is the largest measure of d and c?

7. If we call a 1, what is the name of each of the other units?

8. Show the different units and tell the names.

9. What is the ratio of 1 to each of the other units?

10. What is the ratio of each to 1? of $\frac{1}{2}$ of each to $\frac{1}{2}$ of 1?

11. What is the ratio of 1 bu. to 1 pk.? of $\frac{1}{2}$ bu. to $\frac{1}{2}$ pk.?

12. What is the ratio of 1 sq. yd. to 1 sq. ft.? of $\frac{1}{4}$ sq. yd. to $\frac{1}{4}$ sq. ft.?

13. What is the ratio of 1 yd. to 1 ft.? of $\frac{1}{2}$ yd. to $\frac{1}{2}$ ft.?

14. What is the ratio of any part of 1 yd. to a corresponding part of 1 ft.?

15. If the ratio of the rectangle b to the rectangle a is x, what is the ratio of any part of the rectangle a to a corresponding part of the rectangle b?

16. To paint a blackboard equal to the rectangle a costs \$6; to paint one equal to the rectangle b costs \$5. What is the ratio of b to a?

17. What is the ratio of the time required to paint $\frac{1}{2}$ of b to the time required to paint $\frac{1}{2}$ of a?

18. Observe the above units. If a is 1, what is b?

19. What is the ratio of $\frac{5}{6}$ to each? of each to $\frac{5}{6}$?

20. What is the ratio of $\frac{2}{3}$ to each of the others? of $\frac{1}{3}$? of $\frac{1}{6}$?

21. Draw lines having the same ratios as the solids above. Show the line whose ratio to d equals the ratio of e to d; of c to b; of f to a.

22. The line a can be separated into how many parts, each equal to f? the line c?

23. The sum of what two lines equals the longest line?

24. Write sentences similar to this: the sum of $\frac{1}{6}$ and $\frac{1}{3}$ equals $\frac{1}{2}$.

25. Make sentences like this: $\frac{1}{2}$ equals $\frac{3}{2}$ of $\frac{1}{3}$.

26. Look at the $\frac{1}{6}$. What part of $\frac{1}{3}$ do you find in $\frac{1}{6}$?

27. What part of $\frac{1}{2}$ in $\frac{1}{3}$? what part of the $\frac{2}{3}$ in $\frac{1}{2}$?

28. What part of $\frac{5}{6}$ in each unit?

29. What is the ratio of 6 to 1? of 1 to $\frac{1}{6}$? of 1 to 6? of $\frac{1}{6}$ to 1?

30. What is the ratio of 2 to 3? of $\frac{1}{3}$ to $\frac{1}{2}$? of 3 to 2? of $\frac{1}{2}$ to $\frac{1}{3}$?

31. What part of $\frac{1}{3}$ is the largest common measure of $\frac{1}{3}$ and $\frac{1}{2}$? How many of these measures in $\frac{1}{3}$? in $\frac{1}{2}$? What part of 1 is the largest common measure of $\frac{1}{3}$ and $\frac{1}{2}$?

32. What is $\frac{1}{2}$ of $\frac{1}{3}$? What is $\frac{1}{3}$ of $\frac{1}{2}$? $\frac{2}{3}$ of $\frac{1}{2}$ equals what? $\frac{1}{2}$ of $\frac{2}{3}$ equals what? What, then, is true of $\frac{2}{3}$ of $\frac{1}{2}$ and $\frac{1}{2}$ of $\frac{2}{3}$?

33. $\frac{2}{3}$ of $\frac{1}{2}$ of 1 yard is used for a croquet ground, and $\frac{1}{2}$ of $\frac{2}{3}$ of it is a flower bed. What is the relative size of the flower bed and of the croquet ground?

1. What is the ratio of 1 to $\frac{5}{6}$? to $\frac{1}{2}$? to $\frac{2}{3}$?
2. What is the ratio of $1\frac{1}{2}$ to $\frac{5}{6}$? to $\frac{1}{2}$? to $\frac{2}{3}$?

3. What is the ratio of 1 to $1\frac{1}{8}$? to $1\frac{1}{2}$? to $1\frac{1}{3}$? to $1\frac{5}{8}$?

4. What is the ratio of $\frac{1}{2}$ of x to $\frac{1}{3}$ of x?

5. What is the ratio of $\frac{5}{6}$ of x to $\frac{1}{2}$ of x?

6. What is the ratio of $\frac{3}{4}$ of 7 to $\frac{1}{2}$ of 7?

7. What is the ratio of $\frac{2}{3}$ of $19\frac{1}{2}$ to $\frac{1}{2}$ of $19\frac{1}{2}$?

8. What is the ratio of 7 to $\frac{3}{4}$ of 7? of 9 to $\frac{2}{3}$ of 9?

9. What is the ratio of x to $\frac{x}{2}$?

10. What is the ratio of $\frac{1}{6}$ to $\frac{1}{2}$ of $\frac{1}{6}$? of $\frac{1}{4}$ to $\frac{1}{3}$ of $\frac{1}{4}$?

11. What is the ratio of $\frac{1}{4}$ to $\frac{3}{4}$ of $\frac{1}{4}$? of $\frac{1}{5}$ to $\frac{3}{7}$ of $\frac{1}{5}$?

12. Make problems similar to the following: If a slate costs $\$\frac{1}{3}$ and a book $\$\frac{1}{2}$, what is the cost of both?

13. How many strips of carpet $\frac{1}{2}$ yd. long can be cut from a strip 6 yd. long?

14. At $z¢$ for $\frac{1}{2}$ doz. oranges, $\frac{1}{6}$ doz. costs what part of $z¢$? What is the ratio of the cost of $\frac{1}{6}$ doz. to the cost of $\frac{2}{3}$ doz.?

15. x equals the cost of $\frac{1}{2}$ barrel of flour. What part of x equals the cost of $\frac{3}{4}$ of a barrel?

16. Mrs. Jones paid $\frac{2}{3}$ of $\frac{1}{2}$ of her money for a dress and $\frac{1}{2}$ of $\frac{2}{3}$ of it for a cloak. What was the ratio of the cost of the dress to the cost of the cloak? What part of her money did she spend?

17. At $36¢$ a dozen, what is the cost of $\frac{1}{2}$ of $\frac{1}{6}$ of a dozen oranges? What is the cost of one orange? What is the ratio of $\frac{1}{12}$ of a unit to $\frac{1}{2}$ of $\frac{1}{6}$ of the unit?

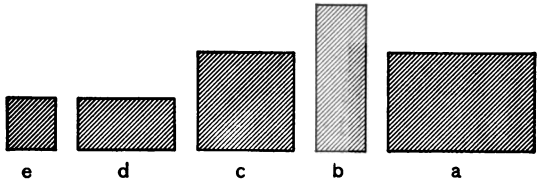
18. The ratio of a larger unit to a smaller is 70. What is the ratio of $\frac{1}{3}$ of the larger to $\frac{1}{3}$ of the smaller?

19. What is the ratio of a rectangle 3 by 2 to a rectangle 2 by 2? What is the ratio of $\frac{1}{2}$ of the larger rectangle to $\frac{1}{2}$ of the smaller? If an amount equal to $\frac{1}{3}$ of the larger rectangle were added to the smaller rectangle, what would then be the ratios of the rectangles?

20. What is the ratio of 8 oz. to 1 lb.? of $\frac{1}{2}$ of 8 oz. to $\frac{1}{2}$ lb.?

21. What is the ratio of $\frac{2}{3}$ of a cord of wood to $\frac{2}{3}$ of a cord? of $\frac{1}{3}$ cord to $\frac{1}{2}$ cord?

22. A man bought a farm for $\$x$ and sold $\frac{2}{3}$ of it for an amount equal to what he paid for it. What was the ratio of the amount he received for $\frac{1}{3}$ of the farm to the amount he paid for the farm? of the amount he received for $\frac{1}{3}$ of the farm to the amount he paid for $\frac{1}{2}$ of it?



1. Draw these rectangles on the blackboard, making e 1 sq. ft.

2. Write all the ratios that you see.

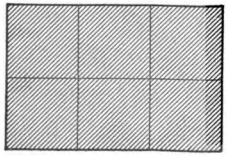
3. If a is 1, what is each of the other units?

4. Compare 1 with each of the others. Compare $\frac{1}{2}$ with each. Compare $\frac{2}{3}$ with each. Compare $\frac{1}{3}$ with each.

5. If c is 1, what is the name of each of the others?

6. Learn the names $1\frac{1}{2}$, $\frac{3}{4}$, 1, $\frac{1}{2}$, and $\frac{1}{4}$. Compare each with the other four.

1. What is the largest equal part of the unit that you see? Name all the equal parts that you see.



2. Compare the unit with each of the parts. Compare $\frac{5}{6}$ with each of the other parts. Compare $\frac{2}{3}$ with each. Compare $\frac{1}{2}$ with each. Compare $\frac{1}{3}$ with each.

3. $\frac{1}{3}$ equals what part of $\frac{1}{2}$? of $\frac{5}{6}$?

4. What part of $\frac{5}{6}$ do you find in $\frac{1}{3}$? in $\frac{1}{2}$? in $\frac{2}{3}$?

5. What is the ratio of 1 to $\frac{2}{3}$? of 1 to $1\frac{1}{3}$?

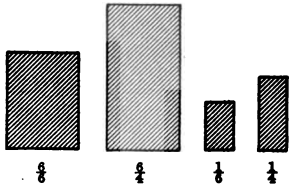
6. Show me $\frac{2}{3}$ of the unit; the $\frac{3}{3}$; the $\frac{6}{3}$.

7. Look at the rectangle. What two equal units in $\frac{6}{3}$? What three equal units?

1. What is the ratio of $\frac{3}{3}$ to $\frac{3}{4}$? of $\frac{4}{3}$ to $\frac{4}{4}$? of $\frac{1}{3}$ to $\frac{1}{4}$?

2. What is the ratio of $\frac{5}{3}$ to $\frac{5}{3}$? of $\frac{5}{5}$ to $\frac{5}{5}$? of $\frac{1}{5}$ to $\frac{1}{5}$?

3. What is the ratio of $\frac{2}{2}$ to $\frac{2}{3}$? of $\frac{3}{2}$ to $\frac{3}{3}$? of $\frac{1}{2}$ to $\frac{1}{3}$?



4. What is the ratio of $\frac{7}{7}$ to $\frac{7}{6}$? of $\frac{6}{7}$ to $\frac{6}{6}$? of $\frac{5}{7}$ to $\frac{5}{6}$? of $\frac{4}{7}$ to $\frac{4}{6}$? of $\frac{3}{7}$ to $\frac{3}{6}$? of $\frac{2}{7}$ to $\frac{2}{6}$? of $\frac{1}{7}$ to $\frac{1}{6}$?

5. x is the ratio of $\frac{4}{6}$ of the distance to $\frac{4}{4}$ of the distance traveled in an hour. What is the ratio of $\frac{1}{6}$ of the distance to $\frac{1}{4}$ of the distance?

6. $\frac{6}{5}$ is the ratio of the weight of a package of sugar to a package of coffee. What is the ratio of two packages, one weighing $\frac{1}{5}$ as much as the sugar and the other $\frac{1}{5}$ as much as the coffee?

7. $\frac{6}{7}$ of the unit a equals the unit b. $\frac{1}{7}$ of the unit a equals what part of the unit b? What part of a is an exact measure of both a and b? what part of b?

8. The sum of $\frac{1}{7}$ of a and $\frac{1}{6}$ of b equals what part of a? of b?

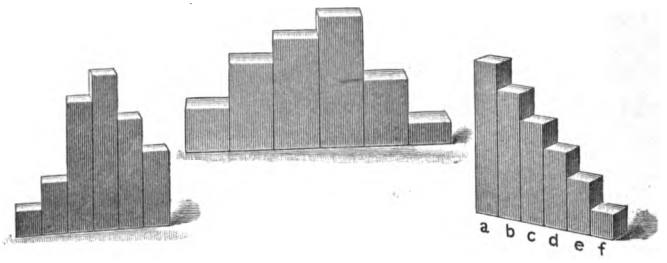
9. If $\frac{1}{2}$ of the width of a room equals $\frac{1}{3}$ its length, what is the ratio of the length to the width?

10. Make and answer similar problems: The ratio of two rectangles is $\frac{3}{4}$. What part of the smaller is an exact measure of both? of the larger?

11. The ratio of 2 to 3 equals the ratio of $\frac{1}{3}$ to what?

12. The ratio of 6 to 5 equals the ratio of $\frac{1}{5}$ to what?

"Advance in representativeness of thought makes possible advance in abstractness." — HERBERT SPENCER.



1. Select solids that represent 1, $\frac{5}{6}$, $\frac{2}{3}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$ and study until the relations of these units can be given readily.

2. Select other sets having the same ratios and master the relations.

3. Make similar sentences: If $\frac{1}{3}$ weighs 13 lb., $\frac{1}{2}$ weighs $\frac{3}{2}$ of 13 lb. If the cost of $\frac{2}{3}$ of 1 bu. of potatoes is 80¢, the cost of $\frac{1}{2}$ bu. is $\frac{3}{4}$ of 80¢.

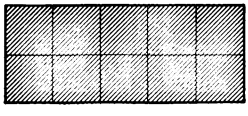
1. Draw a rectangle and separate it into two equal parts.

2. Separate each half into thirds. $\frac{1}{3}$ of $\frac{1}{2}$ equals what part of a rectangle?

3. Separate each 6th into halves. $\frac{1}{2}$ of $\frac{1}{6}$ equals what?

4. Show the two largest equal parts of the rectangle. Show the three largest equal parts. Show the six largest equal parts. Show $\frac{1}{6}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$, and the 1.

1. Draw a rectangle and separate it into 10 equal parts. Two of these parts equal what part of the rectangle?



2. Show me the $\frac{1}{5}$ of a rectangle. Show the 1; the $\frac{4}{5}$; the $\frac{3}{5}$; the $\frac{2}{5}$; the $\frac{1}{5}$.

3. Compare each with the other four.

4. Compare 1 with $1\frac{1}{5}$; with $\frac{4}{5}$; with $\frac{3}{5}$; with $\frac{2}{5}$.

5. What is the ratio of $\frac{1}{5}$ to $\frac{1}{2}$ of $\frac{1}{5}$? of $\frac{1}{7}$ to $\frac{1}{2}$ of $\frac{1}{7}$?

6. What is the ratio of 5 to 1? of 1 to $\frac{1}{5}$?

7. The largest common measure of $\frac{6}{8}$ and $\frac{1}{2}$ of $\frac{6}{8}$ equals what part of the rectangle?

8. Make sentences similar to the following: The sum of $\frac{3}{5}$ and $\frac{1}{10}$ equals $\frac{7}{10}$ of the rectangle. $\frac{1}{3}$ of $\frac{3}{5}$ of the rectangle equals $\frac{3}{5}$ of $\frac{1}{2}$ of the rectangle.

9. What is the ratio of $\frac{5}{5}$ to $\frac{5}{10}$? of $\frac{3}{5}$ to $\frac{2}{10}$? of $\frac{1}{5}$ to $\frac{1}{10}$?

10. What is the ratio of $\frac{3}{2}$ to $\frac{3}{6}$? of $\frac{1}{2}$ to $\frac{1}{6}$?

11. What is the ratio of $\frac{7}{7}$ to $\frac{7}{21}$? of $\frac{1}{7}$ to $\frac{1}{21}$? of a to $\frac{a}{3}$? of $\frac{a}{2}$ to $\frac{1}{2}$ of $\frac{a}{3}$?

1. If 6 doz. peaches cost $\$\frac{3}{4}$, how many dozen can be bought for \$1?

$$\frac{4 \cdot \cancel{6}^{2}}{\cancel{3}_{1}} = 8.$$

Read: Four thirds of 6 equals the number of dozen that can be bought for \$1.

2. Make and express many comparisons, thus: If 9 cost $40\not{c}$, $\frac{5}{9}$ of 9 can be bought for \$1. Why? If 4 can be bought for $\$\frac{1}{7}$, $\frac{7}{2}$ of 4 can be bought for \$1. If in $\frac{8}{3}$ of a box there are 9 doz., there are $\frac{3}{8}$ of 9 doz. in a box.

3. What is the ratio of \$1 to $\$\frac{3}{5}$? What, then, is the ratio of \$4 to $\$\frac{3}{5}$? $\frac{5}{2}$ is the ratio of \$1 to what? $4 \cdot \frac{5}{2}$ is the ratio of \$4 to what?

4. If the cost of 3 pk. of potatoes is $\$\frac{3}{5}$, how many pecks can be bought for \$4?

$$\frac{\cancel{4}^{2} \cdot 5 \cdot 3}{\cancel{2}_{1}} = 30.$$

What is the ratio of \$1 to $\$\frac{3}{5}$? What, then, equals the number that can be bought for \$1? for \$4?

What is the ratio of 30 pk. to 3 pk.? of \$4 to $\$\frac{3}{5}$?

What is the ratio of 3 pk. to 30 pk? of $\$\frac{3}{5}$ to \$4?

5. If 30 pk. of potatoes cost \$4, how many pecks can be bought for $\$\frac{3}{5}$? What part of 30 pk. can be bought for \$1? What part, then, of $\frac{1}{4}$ of 30 pk. can be bought for $\$\frac{3}{5}$?

6. At $\$\frac{3}{5}$ each, how many cakes can be bought for \$6?

7. At $\$\frac{3}{4}$ a yard, how many yards of merino can be bought for \$8?

8. At $\$\frac{4}{5}$ a bushel, how many bushels of potatoes can be bought for \$6? for \$7?

9. A boy earns $\$\frac{4}{5}$ in 1 day. In how many days does he earn \$8?

10. A man mows $\frac{3}{7}$ of an acre of grass in 1 hour. In how many hours does he mow 5 acres?

11. At $\$\frac{3}{8}$ a yard, how many yards of silk can be bought for \$3?

12. At $12\frac{1}{2}¢$ each, how many collars can be bought for \$3?

Practice thinking and writing the ratios in the following until you can do both without hesitation. Write the ratios, nothing more. *Ex.* Write $\frac{3}{2}$; not $\frac{2}{3}$ is the ratio of 1 yd. to 2 ft.

1. What is the ratio of 1 yd. to 2 ft.? to $1\frac{1}{2}$ ft.? to 4 ft.? to $1\frac{1}{3}$ yd.? to $2\frac{1}{2}$ yd.? to 7 ft.? to 1 yd.? to 2 ft.?

2. What is the ratio of 1 bu. to 3 pk.? to 4 pk.? to 5 pk.? to $1\frac{1}{2}$ bu.? to $1\frac{1}{3}$ bu.? to 17 pk.? to $2\frac{1}{2}$ bu.?

3. What is the ratio of 1 sq. yd. to 9 sq. ft.? to 7 sq. ft.? to 6 sq. ft.? to 10 sq. ft.? to $1\frac{1}{3}$ sq. ft.?

4. What is the ratio of a nickel to 3¢? to 4¢? to 7¢? to 9¢? to 10¢?

5. What is the ratio of 1 lb. to 8 oz.? to 12 oz.? to 14 oz.? to 4 oz.? to 20 oz.? to 6 oz.? to $\frac{3}{4}$ lb.? to $\frac{7}{8}$ lb.? to $\frac{2}{3}$ lb.? to $\frac{3}{8}$ lb.? to $1\frac{1}{2}$ lb.? to 17 lb.?

6. What is the ratio of 1 cu. yd. to 27 cu. ft.? to 18 cu. ft.? to 24 cu. ft.? to $1\frac{1}{2}$ cu. ft.?

7. What is the ratio of a 2-in. sq. to 3 sq. in.? to 5 sq. in.? to 7 sq. in.?

8. What is the ratio of 1 yd. to $\frac{3}{4}$ yd.? to $\frac{2}{3}$ yd.? to $\frac{5}{7}$ yd.? to $\frac{5}{8}$ yd.? to $1\frac{1}{2}$ yd.? to $3\frac{2}{3}$ yd.?

9. What is the ratio of \$1 to \$$\frac{3}{4}$? to \$$\frac{5}{8}$? to \$2$\frac{1}{2}$? to \$1$\frac{3}{4}$?

10. What is the ratio of \$1 to \$1.37$\frac{1}{2}$? to \$.37$\frac{1}{2}$? to \$.62$\frac{1}{2}$? to \$.87$\frac{1}{2}$? to \$.12$\frac{1}{2}$? to \$1.37$\frac{1}{2}$?

11. What is the ratio of \$1 to 16$\frac{2}{3}$¢? to 66$\frac{2}{3}$¢? to 83$\frac{1}{3}$¢?

12. What is the ratio of \$1 to 20¢? to 60¢? to 80¢?

13. What is the ratio of 1 to $\frac{3}{4}$? to $\frac{2}{3}$? to $\frac{4}{5}$? to 1$\frac{1}{3}$? to $\frac{3}{5}$? to 1$\frac{1}{2}$? to $\frac{2}{3}$? to $\frac{4}{5}$? to 1$\frac{1}{5}$? to $\frac{6}{5}$? to 1$\frac{1}{4}$?

14. What is the ratio of 1 to $\frac{5}{3}$? to $\frac{7}{5}$? to $\frac{9}{4}$? to $\frac{8}{5}$? to $\frac{11}{5}$? to 3$\frac{1}{3}$? to 17$\frac{1}{2}$? to 18$\frac{1}{3}$? to 23$\frac{2}{3}$?

15. What is the ratio of 1 to $\frac{5}{4}$? to $\frac{7}{5}$? to $\frac{3}{2}$? to $\frac{5}{3}$? to $\frac{9}{5}$? to $\frac{5}{3}$? to $\frac{8}{12}$? to $\frac{12}{17}$? to $\frac{13}{18}$? to $\frac{19}{12}$? to $\frac{13}{19}$?

16. What is the ratio of 1 to .3? to .7? to .5? to .2? to .8? to 1.2? to 1.5? to 7.5? to 2.3?

1. If the cost of 2 yd. of cloth is \$$\frac{3}{4}$, how many yards can be bought for \$5?

$\frac{5 \cdot 4 \cdot 2}{3} = 13\frac{1}{3}$. What is the ratio of \$1 to \$$\frac{3}{4}$? What, then, equals the amount of cloth that can be bought for \$1? for \$5?

2. What is the ratio of \$5 to \$$\frac{3}{4}$? of 13$\frac{1}{3}$ yd. to 2 yd.?

3. What is the ratio of \$$\frac{3}{4}$ to \$5? of 2 yd. to 13$\frac{1}{3}$ yd.?

4. If 13$\frac{1}{3}$ yd. of cloth cost \$5, how many yards can be bought for \$$\frac{3}{4}$?

$\frac{\overset{2}{\cancel{3}} \cdot \cancel{40}}{\underset{1}{\cancel{4}} \cdot \cancel{5} \cdot \cancel{3}} = 2$. What is the ratio of \$1 to \$5? What, then, equals the amount of cloth that can be bought for \$1?

5. The cost of 2 yd. of cloth is \$$\frac{3}{4}$. What is the cost of 13$\frac{1}{3}$ yd.?

6. If 13$\frac{1}{3}$ yd. cost \$5, what is the cost of 2 yd.?

$\frac{\overset{1}{\cancel{2}} \cdot 3 \cdot \cancel{5}}{\underset{4}{\cancel{40}}} = \frac{3}{4}$. What is the ratio of 1 yd. to 13$\frac{1}{3}$ yd.? What, then, equals the cost of 1 yd.? of 2 yd.?

1. Make similar sentences:

If 7 can be bought for $12\frac{1}{2}¢$, $3 \cdot 8 \cdot 7$'s can be bought for \$3. Why?

What is the ratio of \$1 to $12\frac{1}{2}¢$?

What, then, equals the number that can be bought for \$1? for \$3?

2. Make sentences like this:

If in $\frac{2}{7}$ of a bolt there are 10 yd., $\frac{5 \cdot 7 \cdot 10}{2}$ = the number of yards in 5 bolts.

3. Make sentences like this:[1]

If 3 boxes of oranges can be bought for $\$3\frac{1}{2}$ ($\$\frac{7}{2}$), $\frac{14 \cdot 2 \cdot 3}{7}$ equals the number of boxes that can be bought for \$14.

4. Pupils question thus: If 3 doz. plates can be bought for $\$4\frac{1}{2}$, how many dozen can be bought for \$15?

Other pupils write: $\frac{15 \cdot 2 \cdot 3}{9}$.

5. Make sentences like this: $8 \cdot \frac{3}{2}$ is the ratio of 8 bu. to $\frac{3}{2}$ of a bushel.

6. What is the ratio of 9 to $\frac{3}{5}$? What is the ratio of 1 to $\frac{3}{5}$? What, then, is the ratio of 9 to $\frac{3}{5}$?

7. Practice writing the ratios of the following until you can do so readily. *Ex.* What is the ratio of 9 to $\frac{3}{5}$?

Ans. $\frac{9 \cdot 5}{3}$.

8. What is the ratio of 2 to $\frac{3}{7}$? of 5? of 9? of 12?

9. What is the ratio of \$7 to $\$\frac{5}{3}$? of \$2? of \$12? of \$17?

10. What is the ratio of 3 bu. to $\frac{2}{3}$ bu.? of 7 bu.? of 11 bu.?

[1] If pupils cannot do this work with reasonable rapidity, give work which they can do, and approach this through growing power. (See note, p. 73, "Elementary Arithmetic.")

11. What is the ratio of 5 ft. to $\frac{4}{5}$ ft.? of 12 ft.? of 13 ft.?

12. What is the ratio of 2 to $1\frac{5}{7}$? of 5? of 29? of 74?

13. What is the ratio of 6 to $1\frac{3}{7}$? of 26? of 39? of $\frac{1}{2}$?

14. What is the ratio of 9 to $1\frac{7}{3}$? of 5? of 11? of $1\frac{1}{2}$?

15. What is the ratio of 17 to $1\frac{3}{5}$? of 38? of 7? of $2\frac{1}{4}$?

16. What is the ratio of 18 to $1\frac{9}{5}$? of 16? of 144? of $\frac{3}{7}$?

17. Dictate problems thus: What is the ratio of 100 to $\frac{2}{25}$?

Pupils write: $\frac{100 \cdot 25}{2}$ = the ratio of 100 to $\frac{2}{25}$.

1. If there are x qt. in $\frac{3}{5}$ pk., how many x qt. in 8 pk.?

2. If there are 6 qt. in $\frac{3}{4}$ pk., how many qt. in 12 pk.?

$\frac{12 \cdot 4 \cdot 6}{3}$ = the number of qt. in 12 pk.

By what comparisons and inferences was the above statement obtained?

3. There are 10 in. in $\frac{5}{6}$ ft. How many inches are there in $12\frac{1}{2}$ ft.?

4. There are 14 oz. in $\frac{7}{8}$ lb. In 9 lb. how many ounces?

5. There are 6 sq. ft. in $\frac{2}{3}$ of 1 sq. yd. $\frac{3 \cdot 3 \cdot 6}{2}$ equals the number of square feet in how many square yards?

6. There are 21 cu. ft. in $\frac{7}{9}$ cu. yd. How many $\frac{2}{7}$ of 21 cu. ft. in 15 cu. yd.?

7. There are 12 cu. ft. in $\frac{4}{5}$ cu. yd. How many cubic feet in 17 cu. yd.?

8. If there are 48 cu. ft. in $\frac{8}{9}$ cd., how many cubic feet in 7 cd.?

Remark. — Let pupils write similar problems. Call on other pupils to give statements, indicating their solution as in problem 2.

1. What is the ratio of $\frac{3}{5}$ to $\frac{3}{5}$? of $\frac{1}{2}$ to $\frac{1}{3}$? of $\frac{2}{6}$ to $\frac{2}{5}$? of $\frac{1}{6}$ to $\frac{1}{5}$? of $\frac{6}{4}$ to $\frac{6}{6}$? of $\frac{1}{4}$ to $\frac{1}{6}$?

2. Cut two rectangles whose ratio equals the ratio of $\frac{6}{4}$ to $\frac{6}{6}$; of $\frac{1}{4}$ to $\frac{1}{6}$.

3. The ratio of $\frac{6}{4}$ to $\frac{6}{6}$ equals the ratio of what part of $\frac{6}{4}$ to $\frac{1}{6}$?

4. The ratio of $12a$ to a equals the ratio of $\frac{12a}{8}$ to what part of a?

Practice expressing the ratio of a to b, and of b to a at sight. *Ex.* $\frac{3 \cdot 5}{2}$ is the ratio of a to b; $\frac{2 \cdot 1}{5 \cdot 3}$ is the ratio of b to a.

	a	b	a	b
1.	3	$\frac{2}{5}$	6	$\frac{2}{5}$
2.	7	$\frac{5}{4}$	7	$\frac{8}{9}$
3.	4	$\frac{5}{8}$	16	$\frac{2}{4}$
4.	10	$\frac{5}{100}$	13	$\frac{8}{7}$
5.	4	$\frac{5}{4}$	$\frac{2}{3}$	$\frac{3}{4}$

1. If x is the amount that can be bought for \$6, what is the amount that can be bought for $\$\frac{2}{7}$?

$\frac{2 \cdot x}{7 \cdot 6} = \frac{x}{21}$. What equals the amount that can be bought for \$1? for $\$\frac{2}{7}$?
3

2. What is the ratio of $\$\frac{2}{5}$ to \$7?

$\frac{2 \cdot 1}{5 \cdot 7}$ = the ratio of $\$\frac{2}{5}$ to \$7.

What is the ratio of \$1 to \$7? What, then, is the ratio of $\$\frac{2}{5}$ to \$7?

What is the ratio of —

3. $\frac{3}{4}$ to 4?
4. $\frac{2}{3}$ to 7?
5. $\frac{3}{5}$ to 5?
6. $\frac{8}{3}$ to 6?
7. $2\frac{1}{3}$ to 11?
8. .7 to 8?
9. .3 to 5?
10. 1.2 to 13?
11. $\frac{2}{7}$ to $2\frac{1}{2}$?
12. $\frac{3}{11}$ to 15?
13. $\frac{7}{4}$ to 14?
14. $\frac{8}{9}$ to $17\frac{1}{2}$?

1. What is the ratio of 3 sq. ft. to $1\frac{1}{3}$ sq. ft.?

$\frac{3 \cdot 3}{4} = 2\frac{1}{4}$. What is the ratio of 1 sq. ft. to $1\frac{1}{3}$ sq. ft.? What, then, is the ratio of 3 sq. ft. to $1\frac{1}{3}$ sq. ft.?

What is the ratio of —

2. 7 sq. ft. to $2\frac{1}{4}$ sq. ft.? 12 sq. ft. to $4\frac{2}{7}$ sq. ft.?
3. 6 sq. ft. to $3\frac{1}{8}$ sq. ft.? .7 sq. ft. to .7 sq. ft.?
4. 9 sq. ft. to $2\frac{2}{4}$ sq. ft.? $3\frac{2}{7}$ sq. ft. to $7\frac{1}{2}$ sq. ft.?
5. 12 sq. ft. to $3\frac{2}{3}$ sq. ft.? $13\frac{1}{7}$ sq. ft. to $7\frac{1}{2}$ sq. ft.?
6. $2\frac{1}{2}$ sq. ft. to $2\frac{1}{3}$ sq. ft.? $47\frac{1}{2}$ sq. ft. to $24\frac{2}{3}$ sq. ft.?

What is the ratio of —

7. 8 cu. ft. to $2\frac{1}{2}$ cu. ft.? 12 cu. ft. to 17 cu. ft.?
8. 12 cu. ft. to $2\frac{5}{8}$ cu. ft.? .5 cu. ft. to $14\frac{1}{4}$ cu. ft.?
9. $2\frac{1}{4}$ cu. ft. to $7\frac{3}{4}$ cu. ft.? $\frac{2}{3}$ cu. ft. to $14\frac{1}{4}$ cu. ft.?
10. What is the ratio of a 2-in. sq. to $\frac{3}{4}$ sq. in.?

$\frac{4 \cdot 4}{3} = 5\frac{1}{3}$.

11. What is the ratio of a 3-in. sq. to $\frac{3}{4}$ sq. in.?
12. What is the ratio of a 4-in. sq. to $\frac{3}{4}$ sq. in.?
13. What is the ratio of a 6-in. sq. to $\frac{3}{4}$ sq. in.?
14. What is the ratio of a 4-in. sq. to $\frac{2}{7}$ sq. in.?
15. What is the ratio of a 6-in. sq. to $\frac{2}{3}$ sq. in.?
16. What is the ratio of 1 sq. ft. to $\frac{2}{3}$ of a 6-in. sq.?
17. What is the ratio of a 2-in. cube to $\frac{5}{8}$ cu. in.?
18. What is the ratio of a 3-in. cube to $\frac{3}{4}$ cu. in.?

When graduates of high schools and normal schools assert that the distance around a rectangle containing 3 rows of 7 sq. ft. is 21 sq. ft.; that a box required to hold a set of 3-in. cubes should be 3 times the size of a box required to hold a similar set of 1-in. cubes; and when these instances are not exceptional but typical, does it not seem that contact with realities in their *mathematical aspect* is a necessity?

When it is equally common for pupils who can tell that the ratio of a to b is x to be unable to tell that the ratio of $5a$ to $5b$

is x, or $\frac{1}{3}$ of a to $\frac{1}{3}$ of b is x, is it not fair to infer that the presentation is restricting rather than promoting mental action, that it is preventing seeing things as they are?[1]

For a lesson write five problems and questions on each, similar to the following:

1. If the cost of 2 doz. roses is $\$\frac{1}{8}$, how many can be bought for \$14?

$$\frac{\cancel{14}^{2} \cdot 8 \cdot 2}{\cancel{7}} = 32.$$

What equals the number of dozen that can be bought for \$1? for \$14?

2. If the cost of 6 qt. of berries is $\$\frac{3}{4}$, how many quarts can be bought for \$4?

$$\frac{\cancel{4}^{2} \cdot 3 \cdot 6}{\cancel{2}} = 36.$$

(*a*) The cost of 36 qt. is \$4. How many quarts can be bought for $\$\frac{3}{4}$?

(*b*) The cost of 36 qt. is \$4. What is the cost of 6 qt.?

(*c*) The cost of 6 qt. is $\$\frac{3}{4}$. What is the cost of 36 qt.?

Remark.— If you observe the second problem, you will see that it suggests the questions (*a*), (*b*), and (*c*). Ask similar questions on each of the following:

3. If the cost of 5 yd. of cloth is $\$\frac{3}{8}$, how many yards can be bought for \$7?

4. If a 5-ft. staff casts a shadow $\frac{3}{8}$ of a rod long, what is the length of the shadow cast by a staff 7 ft. high?

5. If the cost of 3 lb. of coffee is $\$\frac{5}{8}$, how many pounds can be bought for \$4?

[1] "Whenever I went far enough, I touched geometrical bottom."— PROFESSOR SYLVESTER.

"The higher processes of mind in mathematics lie at the very foundation of the subject."— PROFESSOR SYLVESTER.

6. If x bu. of wheat are required to make 3 bbl. of flour, how many bushels are required to make 4 bbl. of flour?

7. At the rate of 69 mi. in $3\frac{1}{2}$ hr., how far does a car run in 8 hr.?

8. x equals the number of miles a car runs in $5\frac{1}{8}$ hr. In what time does the distance it runs equal $\frac{7 \cdot 3 \cdot x}{16}$?

9. Write three problems. Ask three questions on each. (See problem 2.)

1. If 15 yd. of calico can be bought for $\$\frac{3}{4}$, how many yards can be bought for $\$\frac{2}{3}$?

2. What is the ratio of 1 to $\frac{3}{4}$?

The ratio of $\frac{2}{3}$ to $\frac{3}{4}$ equals what part of the ratio of 1 to $\frac{3}{4}$? What is the ratio of $\frac{2}{3}$ to $\frac{3}{4}$?

3. What is the ratio of $\frac{3}{4}$ to $\frac{5}{7}$?

4. Make similar sentences: $\frac{5}{7}$ of $\frac{2}{3}$, or $\frac{1}{3}\frac{0}{5}$, is the ratio of $\frac{5}{7}$ to $\frac{3}{5}$.

Write the ratios of the following, thus:

$$\frac{4 \cdot 8}{5 \cdot 7}, \text{ or } \frac{32}{35} = \text{the ratio of } \tfrac{4}{5} \text{ to } \tfrac{7}{8}.$$

What is the ratio of —

5. $\frac{4}{11}$ to $\frac{8}{5}$?	9. $\frac{3}{8}$ to $\frac{11}{9}$?	13. $\frac{7}{8}$ to $\frac{3}{4}$?
6. $\frac{24}{4}$ to $\frac{7}{2}$?	10. $\frac{7}{6}$ to $\frac{14}{3}$?	14. $1\frac{1}{2}$ to $\frac{2}{7}$?
7. $\frac{11}{12}$ to $\frac{13}{5}$?	11. $\frac{5}{7}$ to 8?	15. $\frac{18}{19}$ to $\frac{9}{38}$?
8. $\frac{2}{7}$ to $\frac{8}{5}$?	12. $\frac{3}{11}$ to 7?	16. $\frac{18}{19}$ to 9?

1. What equals the ratio of $2\frac{1}{5}$ to $3\frac{1}{2}$?

$\frac{\overset{2}{\cancel{14}} \cdot 2}{5 \cdot \cancel{7}} = \frac{4}{5}$. What is the ratio of 1 to $3\frac{1}{2}$? What, then, equals the ratio of $2\frac{1}{5}$, or $\frac{14}{5}$, to $3\frac{1}{2}$?

2. If x yd. of cloth can be bought for $\$3\frac{1}{2}$, how many yards can be bought for $\$2\frac{4}{5}$?

$$\frac{\overset{2}{\cancel{14}}\cdot 2\cdot x}{5\cdot \cancel{7}}=\frac{4x}{5}.$$

What equals the ratio of —

3. $2\frac{1}{8}$ to $6\frac{1}{2}$?
4. $7\frac{3}{8}$ to $9\frac{1}{2}$?
5. $4\frac{1}{7}$ to $9\frac{1}{2}$?
6. $2\frac{4}{9}$ to $3\frac{2}{3}$?
7. $5\frac{1}{2}$ to $2\frac{2}{4}$?
8. $3\frac{3}{4}$ to $7\frac{1}{8}$?
9. $2\frac{1}{3}$ to $3\frac{1}{4}$?
10. $5\frac{1}{2}$ to $7\frac{3}{5}$?
11. $6\frac{2}{3}$ to $9\frac{3}{4}$?
12. $33\frac{1}{8}$ to $51\frac{2}{3}$?
13. $13\frac{2}{3}$ to $17\frac{1}{7}$?
14. $13\frac{2}{3}$ to 5?

1. If the cost of 10 yd. of lace is $\$16\frac{4}{5}$, how many yards can be bought for $\$52\frac{2}{5}$?

Suggestion. — After finding the answer to this question, make and solve three other problems suggested by the question.

2. If a man earns $\$2\frac{2}{3}$ in 1 day, in how many days does he earn $\$13\frac{1}{3}$?

3. If $\frac{5}{6}$ yd. of silk is enough for 3 neckties, $8\frac{1}{3}$ yd. are enough for how many neckties?

4. If a man pays $\$\frac{3}{4}$ a day for his meals, in how many days does he spend $\$16\frac{1}{2}$?

5. If a family uses $\frac{1}{3}$ of a barrel of flour in 1 wk., how long do $5\frac{1}{4}$ barrels last?

6. If the cost of x lb. of coffee is \$7, how many pounds can be bought for $\$7\frac{1}{4}$?

7. At $\$\frac{1}{3}$ a pound, how many pounds of coffee can be bought for \$2? for \$5? for $66\frac{2}{3}$¢?

8. At $\$\frac{3}{4}$ a yard, how many yards of cloth can be bought for \$18?

9. At $\$5\frac{1}{2}$ a cord, how many cords of wood can be bought for \$84?

10. How many pieces of cloth $\frac{5}{6}$ yd. long can be cut from a piece 10 yd. long?

11. How many yards of ribbon at $\$\frac{1}{4}$ each can be bought for $\$\frac{7}{4}$?

12. .1 is the ratio of what to .7? to 2.4?

13. The ratio of .3 to .5 equals the ratio of 3 to what?

14. The ratio of $\frac{5}{8}$ to $\frac{5}{6}$ equals the ratio of what to $\frac{1}{8}$?

15. Alice has a piece of green ribbon 9 yd. in length and a piece of blue 6 yd. in length. She wishes to cut them into the longest equal pieces possible. When cut, what is the ratio of each piece to 9 yd.? of each piece to 6 yd.?

1. What is the ratio of $2\frac{1}{2}$ to .7?

$\frac{5 \cdot \not{10}}{\not{2} \cdot 7} = \frac{25}{7} = 3\frac{4}{7}$. What is the ratio of 1 to .7? What, then, is the ratio of $2\frac{1}{2}$, or $\frac{5}{2}$, to .7?

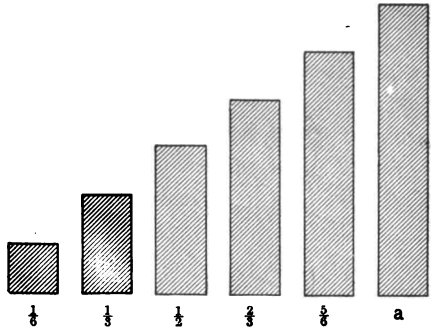
What is the ratio of —

2. $9\frac{1}{2}$ to .3? 9 to .5?
3. $3\frac{1}{3}$ to .8? $2\frac{2}{7}$ to 1.7?
4. $18\frac{1}{2}$ to .7? $\frac{5}{8}$ to .9?
5. $6\frac{2}{5}$ to 1.1? $13\frac{1}{3}$ to 2.5?
6. $7\frac{3}{4}$ to 1.3? $5\frac{3}{7}$ to 4.3?
7. What is the ratio of 1.2 $(1\frac{2}{10})$ to $9\frac{1}{2}$?

$\frac{12 \cdot \not{2}}{\not{10} \cdot 19} = \frac{12}{95}$. What is the ratio of 1 to $9\frac{1}{2}$? What, then, is the ratio of 1.2 to $9\frac{1}{2}$? $\frac{12}{95}$ is the ratio of 1 to what? 1.2 of $\frac{12}{95}$ is the ratio of 1.2 to what?

What is the ratio of —

8. .3 to $9\frac{1}{2}$? .5 to 9?
9. .8 to $3\frac{1}{3}$? 1.7 to $2\frac{2}{7}$?
10. .7 to $18\frac{1}{2}$? .9 to $\frac{5}{8}$?
11. 1.1 to $6\frac{2}{5}$? 2.5 to $13\frac{3}{4}$?
12. 1.3 to $7\frac{3}{4}$? 4.3 to $5\frac{3}{7}$?



1. If a is 1, each of the other units equals what part of 1? Compare each unit with every other unit in the diagram.

2. What is the ratio of $\frac{1}{3}$ to $\frac{1}{2}$ of $\frac{1}{3}$? of $\frac{1}{6}$ to $\frac{1}{2}$ of $\frac{1}{6}$? of $\frac{1}{2}$ to $\frac{2}{3}$ of $\frac{1}{2}$? of $\frac{5}{6}$ to $\frac{2}{3}$ of $\frac{5}{6}$? of $\frac{2}{3}$ to $\frac{1}{4}$ of $\frac{2}{3}$? of $\frac{2}{3}$ to $\frac{1}{5}$ of $\frac{2}{3}$?

3. What is the ratio of $\frac{1}{2}$ to $\frac{1}{3}$ of $\frac{1}{2}$?

4. What is the ratio of 1 to $\frac{1}{3}$ of $\frac{1}{2}$? of $\frac{1}{3}$ of $\frac{1}{2}$ to 1?

5. What is the ratio of $\frac{1}{8}$ to $\frac{1}{4}$ of $\frac{1}{8}$?

6. What, then, is the ratio of 1 to $\frac{1}{4}$ of $\frac{1}{3}$? of $\frac{1}{4}$ of $\frac{1}{8}$ to 1?

7. What is the ratio of 9 to $\frac{1}{2}$ of $\frac{1}{8}$ of 9?

"A great part of the progress of formal human thought . . . has been due to the invention of what we may call short-mind symbols. . . . But it should never be forgotten that the mighty stenophrenic engine of which we here speak, like all machinery, affords us rather a mastery over nature than an insight into it; and for some, unfortunately, the higher symbols of mathematics are merely brambles that hide the living springs of reality.

"Many of the greatest discoveries of science — for example, those of Galileo, Huygens, and Newton — were made without the mechanism which afterwards becomes so indispensable for their development and application." — T. J. McCormack, Open Court, December, 1897.

8. What is the ratio of 2 to $\frac{1}{2}$ of $\frac{1}{8}$ of 2?
9. What is the ratio of 1 to $\frac{1}{2}$ of $\frac{1}{8}$ of 1?
10. What is the ratio of x to $\frac{1}{2}$ of $\frac{1}{8}$ of x?

1. What is $\frac{1}{2}$ of $\frac{1}{3}$ of 1? What is the ratio of $\frac{1}{3}$ to $\frac{1}{2}$ of $\frac{1}{3}$? What, then, is the ratio of 1 to $\frac{1}{2}$ of $\frac{1}{3}$? What, then, is the ratio of $\frac{1}{2}$ of $\frac{1}{3}$ to 1?
2. What part of x is $\frac{1}{2}$ of $\frac{1}{3}$ of x?
3. What is the ratio of x to $\frac{1}{2}$ of $\frac{1}{3}$ of x?
4. What part of an apple is $\frac{1}{3}$ of $\frac{1}{4}$ of it?
5. What is the ratio of a unit to $\frac{1}{3}$ of $\frac{1}{4}$ of the unit?
6. What part of 1 ft. is $\frac{1}{5}$ of $\frac{1}{7}$ of a ft.? Why?
7. What is the ratio of 2 ft. to $\frac{1}{5}$ of $\frac{1}{7}$ of 2 ft.?
8. What is the ratio of x to $\frac{1}{5}$ of $\frac{1}{7}$ of x?
9. What part of a unit is $\frac{1}{2}$ of $\frac{1}{5}$ of the unit? $\frac{1}{3}$ of $\frac{1}{6}$? $\frac{1}{7}$ of $\frac{1}{8}$? $\frac{1}{8}$ of $\frac{1}{9}$? $\frac{1}{10}$ of $\frac{1}{10}$? .1 of .1? $\frac{1}{4}$ of $\frac{1}{25}$?
10. If $\frac{1}{4}$ of $\frac{1}{5}$ is $\frac{1}{20}$ of a unit, what part of a unit is $\frac{1}{4}$ of $\frac{3}{5}$?

"All the materials of intellect are images and symbols, all its processes are operations on images and symbols." — LEWES.

Operations on symbols are barren without the experiences which give them significance. To judge of the ratio of $\frac{2}{3}$ and $\frac{5}{6}$ these symbols should call up such distinct images of the things that their ratio is at once grasped. Until this ratio is seen it cannot furnish a groundwork for conclusions concerning things. To know vaguely the meaning of $\frac{2}{3}$, of $\frac{5}{6}$, and $\frac{5}{4}$, is not enough; we must see $\frac{2}{3}$ and $\frac{5}{6}$ in the relation upon which our inference is based. For example, if a pupil is expected to see that a man who receives \$8 for doing $\frac{2}{3}$ of a piece of work should receive $\frac{5 \cdot \$8}{4}$ for doing $\frac{5}{6}$ of it, he should have had experiences which cause him to so realize the relation of $\frac{5}{6}$ to $\frac{2}{3}$, that $\frac{5 \cdot \$8}{4}$ will at once suggest itself as a necessary relation.

11. If $\frac{1}{4}$ of $\frac{3}{5}$ equals $\frac{3}{20}$ of a unit, what part of a unit is $\frac{5}{4}$ of $\frac{3}{5}$?

12. To what part of a unit is $\frac{2}{5}$ of $\frac{3}{7}$ of it equal?

13. What is $\frac{5}{6}$ of $\frac{7}{8}$? $\frac{4}{7}$ of $\frac{3}{11}$ equals what?

14. What is $\frac{3}{5}$ of $\frac{2}{7}$? What is $\frac{4}{3}$ of $\frac{7}{5}$?

15. What part of 7 is $\frac{4}{11}$ of $\frac{5}{6}$ of 7?

16. What is the ratio of 1 to $\frac{3}{15}$ of $\frac{4}{5}$ of 1?

17. What is the ratio of .9 of $\frac{3}{5}$ of 1 to 1? What is .1 of $\frac{3}{5}$? What, then, is .9 of $\frac{3}{5}$?

18. 1.2 of $\frac{7}{11}$ equals what? What is .1 of $\frac{7}{11}$? What, then, is $\frac{12}{10}$ of $\frac{7}{11}$?

19. Pupils write problems similar to problem 18, and represent answers by drawings.

1. $\frac{7}{12}$ of $\frac{8}{9}$ = ?	$\frac{7}{36}$ of $\frac{96}{8}$ = ?
2. $\frac{5}{12}$ of $\frac{15}{17}$ = ?	$\frac{1}{3}$ of $13\frac{1}{2}$ = ?
3. $\frac{7}{16}$ of $\frac{8}{22}$ = ?	$\frac{1}{4}$ of $21\frac{1}{4}$ = ?
4. $\frac{1}{12}$ of $\frac{4}{13}$ = ?	$\frac{1}{3}$ of $28\frac{1}{4}$ = ?
5. $\frac{5}{9}$ of 21 = ?	$\frac{3}{8}$ of $42\frac{1}{3}$ = ?
6. $\frac{7}{9}$ of $\frac{7}{21}$ = ?	$\frac{7}{9}$ of $37\frac{1}{2}$ = ?
7. $\frac{10}{19}$ of $\frac{38}{16}$ = ?	$\frac{10}{19}$ of $\frac{3}{16}$ = ?
8. $\frac{3}{26}$ of $\frac{39}{4}$ = ?	$\frac{5}{7}$ of $\frac{28}{39}$ = ?
9. .9 of $\frac{30}{17}$ = ?	1.3 of $\frac{8}{7}$ = ?
10. $\frac{3}{5}$ of $8\frac{2}{5}$ = ?	$\frac{7}{4}$ of $15\frac{1}{3}$ = ?

See note, p. 12, "Elementary Arithmetic."
Review the work, pp. 258–260, "Elementary Arithmetic."

1. If x equals the amount of work 4 men can do in a day, $5x$ equals what?

2. A can do $\frac{1}{2}$ of a piece of work in 1 day and B $\frac{1}{3}$ of it. What part of the work can both do in a day?

3. A can mow x acres in $\frac{1}{2}$ of a day, and B can mow x acres in $\frac{1}{3}$ of a day. If both work for a day, what is the ratio of the number of acres mowed to x acres?

4. John can do a piece of work in a day, and James in $\frac{1}{2}$ of a day. If both work together what is the ratio of the work they can do in a day to the work done?

5. If 8 men mow $7\frac{3}{5}$ acres of grass, how much do 3 men mow in the same time? The ratio of 3 to 8 equals the ratio of what to $7\frac{3}{5}$ acres?

6. If 12 men pave $5\frac{1}{8}$ rd. of street in x hr., how much do 5 men pave in x hr.?

$$\frac{5 \cdot \overset{4}{\cancel{12}}}{\underset{3}{\cancel{12}} \cdot 3} = ?$$ What ratios are equal?

7. If for \$60 you can buy $\frac{3}{5}$ of an acre of land, for \$12 you can buy how much?

8. If the cost of $\frac{3}{5}$ of an acre of land is \$60, what is the cost of $\frac{3}{2\,5}$ of an acre?

9. If \$12 is the cost of $\frac{3}{2\,5}$ of an acre of land, what is the cost of $\frac{3}{5}$ of an acre?

10. If $\frac{3}{2\,5}$ of an acre of land costs \$12, how much can be bought for \$60?

11. $\frac{7}{8}$ is the ratio of a larger farm to a smaller. What is the ratio of the amount of money required to buy $\frac{1}{8}$ of the larger to the amount required to buy $\frac{1}{8}$ of the smaller?

12. $\frac{100}{7}$ is the ratio of the money a man borrowed to the interest he paid. What is the ratio of the interest he paid to the money borrowed? If he borrowed \$742, how much interest did he pay?

13. \$400 equals $\frac{100}{10}$ of the interest a man pays in 1 year. How much interest does he pay? What is the ratio of the interest he pays to \$400?

14. A merchant sold goods for a sum equal to $\frac{4}{3}$ of the cost. What was the ratio of the cost to the selling price?

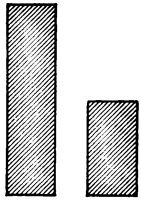
15. What is the cost of $1\frac{1}{2}$ yd. of cloth at $\$\frac{4}{5}$ a yard?

16. At $15\frac{1}{4}¢$ a pound, what is the cost of $\$4\frac{1}{2}$ lb. of beef?

17. If a train runs 125$\frac{3}{7}$ mi. in 6 hr., what is the rate per hour?

18. In each of the above problems state what ratios are equal.

1. What is the ratio of 12 to 6? of $\frac{1}{2}$ of 12 to $\frac{1}{2}$ of 6? of $\frac{1}{6}$ of 12 to $\frac{1}{6}$ of 6? of $\frac{2}{3}$ of 12 to $\frac{2}{3}$ of 6? of $\frac{3}{4}$ of 12 to $\frac{3}{4}$ of 6?



2. What is the ratio of 56 to 14? What is the ratio of $\frac{1}{7}$ of 56 to $\frac{1}{7}$ of 14? What is true of the ratio of 56 to 14 and of $\frac{1}{7}$ of 56 to $\frac{1}{7}$ of 14?

3. What is the ratio of $\frac{1}{8}$ to $\frac{1}{2}$? What is the ratio of $\frac{1}{2}$ of $\frac{1}{8}$ to $\frac{1}{2}$ of $\frac{1}{2}$?

4. How can the ratio of two units be found without a *direct* comparison of the units?

5. How can the ratio of 12 to 15 be found indirectly? What part of each can be found easily? What is $\frac{1}{3}$ of 12? of 15? What is the ratio of 4 to 5? What, then, is the ratio of 12 to 15? $\frac{12}{15}\left|\frac{4}{5}\right.$.

6. $\frac{1}{7}$ of 14 and $\frac{1}{7}$ of 21 are corresponding parts of 14 and 21. Give examples of corresponding parts of other units. The ratio of $\frac{1}{7}$ of 14 to $\frac{1}{7}$ of 21 equals the ratio of what to 21? What is the ratio of $\frac{1}{7}$ of 14 to $\frac{1}{7}$ of 21? What, then, is the ratio of 14 to 21? What ratios are equal? The ratio of the *corresponding parts* of two units equals the ratio of what? The ratio of two *units equals* the ratio of what?

7. What is the ratio of $\frac{1}{8}$ of $\frac{8}{6}$ to $\frac{1}{8}$ of $\frac{8}{3}$? What, then, is the ratio of $\frac{8}{6}$ to $\frac{8}{3}$? Why?

8. What is the ratio of $\frac{1}{4}$ to $\frac{1}{2}$? What, then, is the ratio of $\frac{5}{4}$ to $\frac{5}{2}$? Why?

9. $\frac{3}{4}$ is the ratio of x to y. What, then, is the ratio of $\frac{1}{2}$ of x to $\frac{1}{2}$ of y? of $\frac{1}{3}$ of x to $\frac{1}{3}$ of y? Why?

10. $\frac{2}{3}$ is the ratio of what to 45? $\frac{2}{3}$ is the simplest expression of the ratio of what to 45? $15\left|\frac{30}{45}\right|\frac{2}{3}$.

Express the following ratios in the simplest form:

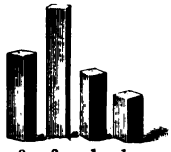
1. $\frac{30}{45}$.	5. $\frac{36}{48}$.	9. $\frac{60}{120}$.	13. $\frac{16}{18}$.
2. $\frac{20}{25}$.	6. $\frac{56}{84}$.	10. $\frac{18}{20}$.	14. $\frac{180}{180}$.
3. $\frac{11}{33}$.	7. $\frac{80}{100}$.	11. $\frac{20}{44}$.	15. $\frac{15}{45}$.
4. $\frac{28}{32}$.	8. $\frac{13}{39}$.	12. $\frac{26}{52}$.	16. $\frac{150}{450}$.

Give at sight the simplest form of the following:

1. $\frac{9}{12}$.	5. $\frac{11}{33}$.	9. $\frac{6}{21}$.	13. $\frac{24}{32}$.
2. $\frac{12}{16}$.	6. $\frac{22}{66}$.	10. $\frac{16}{64}$.	14. $\frac{21}{49}$.
3. $\frac{10}{15}$.	7. $\frac{33}{99}$.	11. $\frac{17}{34}$.	15. $\frac{30}{48}$.
4. $\frac{14}{21}$.	8. $\frac{14}{42}$.	12. $\frac{17}{51}$.	16. $\frac{42}{66}$.

17. $\frac{60}{300}$. 18. .35. 19. $\frac{150}{300}$. 20. .625. 21. .15. 22. .125.

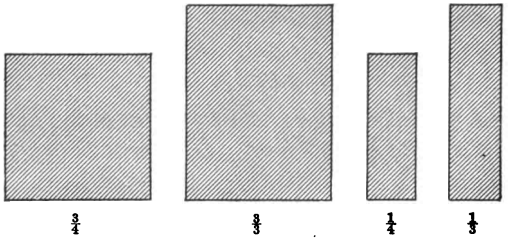
1. Draw $\frac{2}{3}$ and $\frac{3}{4}$. Draw $\frac{1}{3}$ and $\frac{1}{2}$. What is the ratio of $\frac{2}{3}$ to $\frac{3}{4}$? What, then, is the ratio of $\frac{1}{3}$ to $\frac{1}{2}$? Why? What parts are the corresponding parts? What is the ratio of $\frac{2}{3}$ to $\frac{3}{4}$? What, then, is the ratio of $\frac{1}{2}$ to $\frac{1}{3}$?



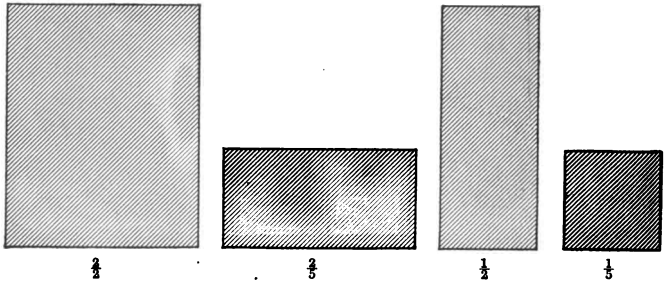
2. Draw $\frac{3}{4}$ and $\frac{3}{5}$. Draw $\frac{1}{4}$ and $\frac{1}{5}$. What is the ratio of the $\frac{3}{5}$ to the $\frac{3}{4}$? What, then, is the ratio of $\frac{1}{5}$ to $\frac{1}{4}$? Why? What is the ratio of $\frac{3}{4}$ to $\frac{3}{5}$? What, then, is the ratio of $\frac{1}{4}$ to $\frac{1}{5}$? Why? What is the ratio of $\frac{1}{2}$ to $\frac{1}{3}$? of $\frac{1}{3}$ to $\frac{1}{2}$?

3. Draw $\frac{4}{4}$ and $\frac{4}{5}$. Draw $\frac{1}{4}$ and $\frac{1}{5}$. Tell what you can about the ratios of these units and of their corresponding parts. What is the ratio of $\frac{5}{4}$ to $\frac{5}{5}$? of $\frac{4}{4}$ to $\frac{4}{5}$? of $\frac{7}{4}$ to $\frac{7}{5}$?

of $\frac{1}{4}$ to $\frac{1}{5}$? What is the ratio of $\frac{4}{5}$ to $\frac{4}{4}$? What, then, is the ratio of $\frac{1}{5}$ to $\frac{1}{4}$?



4. Draw units having the relative size of $\frac{2}{2}$ and $\frac{2}{5}$; of $\frac{1}{2}$ and $\frac{1}{5}$.



5. What is the ratio of $\frac{2}{2}$ to $\frac{2}{5}$? of $\frac{1}{2}$ to $\frac{1}{5}$? of $\frac{2}{5}$ to $\frac{2}{2}$? of $\frac{1}{5}$ to $\frac{1}{2}$?

6. What is the ratio of $\frac{1}{2}$ to $\frac{1}{3}$? to $\frac{1}{4}$? to $\frac{1}{5}$? to $\frac{1}{6}$? to $\frac{1}{8}$?

7. What is the ratio of $\frac{1}{3}$ to $\frac{1}{4}$? to $\frac{1}{5}$? to $\frac{1}{6}$? to $\frac{1}{7}$? to $\frac{1}{8}$?

8. What is the ratio of $\frac{1}{4}$ to $\frac{1}{5}$? to $\frac{1}{6}$? to $\frac{1}{7}$? to $\frac{1}{8}$?

9. $\frac{5}{2}$ is the ratio of $\frac{2}{2}$ to $\frac{2}{5}$. Why?

10. Make many similar sentences: $\frac{5}{2}$ is the ratio of $\frac{2}{2}$ to $\frac{2}{5}$. $\therefore$ $\frac{5}{2}$ is the ratio of $\frac{1}{2}$ to $\frac{1}{5}$.

11. What is the ratio of $\frac{8}{7}$ to $\frac{8}{8}$? of $\frac{7}{7}$ to $\frac{7}{8}$? of $\frac{1}{7}$ to $\frac{1}{8}$? Why?

12. Make many similar sentences: $\frac{7}{8}$ is the ratio of $\frac{1}{8}$ to $\frac{1}{7}$.

13. What is the ratio of $\frac{8}{12}$ to $\frac{8}{8}$? of $\frac{1}{12}$ to $\frac{1}{8}$?

14. What is the ratio of $\frac{40}{60}$ to $\frac{60}{60}$?

15. What is the ratio of $\frac{40}{60}$ to $\frac{40}{40}$? What, then, is the ratio of $\frac{1}{60}$ to $\frac{1}{40}$?

16. $\frac{2}{3}$ is the ratio of $\frac{30}{45}$ to $\frac{30}{30}$. Why?

17. Make many similar sentences: $\frac{2}{3}$ is the ratio of $\frac{30}{45}$ to $\frac{30}{30}$. $\therefore$ $\frac{2}{3}$ is the ratio of $\frac{1}{45}$ to $\frac{1}{30}$.

To Teacher. — Train pupils to draw quickly units in different relations. Through such representation the condition of the pupil's mind is revealed. A pupil may learn to *say* that 5 is the ratio of 45 to 9, or $\frac{2}{3}$ the ratio of $\frac{1}{8}$ to $\frac{1}{2}$ without *seeing* these relations. One who perceives mathematical relations will be able to represent the magnitudes compared.

18. What is the ratio of $\frac{9}{12}$ to $\frac{8}{9}$? of $\frac{1}{12}$ to $\frac{1}{9}$? of $\frac{1}{9}$ to $\frac{1}{12}$?

19. Draw units that represent the relative size of $\frac{1}{3}$ and $\frac{1}{6}$; of $\frac{1}{12}$ and $\frac{1}{8}$; of $\frac{1}{6}$ and $\frac{1}{8}$; of $\frac{1}{216}$ and $\frac{1}{13}$; of $\frac{1}{30}$ and $\frac{1}{45}$; of $\frac{1}{8}$ and $\frac{1}{4}$; of $\frac{1}{27}$ and $\frac{1}{9}$; of $\frac{1}{18}$ and $\frac{1}{27}$; of $\frac{1}{6}$ and $\frac{1}{18}$; of $\frac{1}{22}$ and $\frac{1}{44}$; of $\frac{1}{75}$ and $\frac{1}{50}$.

20. Practice making mental pictures of units having the relative size of the following: $\frac{1}{3}$ and $\frac{1}{2}$, $\frac{1}{5}$ and $\frac{1}{2}$, $\frac{1}{6}$ and $\frac{1}{9}$, $\frac{1}{12}$ and $\frac{1}{18}$, $\frac{1}{8}$ and $\frac{1}{4}$, $\frac{1}{7}$ and $\frac{1}{8}$, $\frac{1}{14}$ and $\frac{1}{7}$, $\frac{1}{21}$ and $\frac{1}{14}$, $\frac{1}{15}$ and .1, $\frac{1}{5}$ and .1, $\frac{1}{15}$ and .1, $\frac{1}{24}$ and $\frac{1}{72}$, $\frac{1}{40}$ and $\frac{1}{80}$, $\frac{1}{60}$ and $\frac{1}{40}$.

21. Make similar sentences: The ratio of $\frac{1}{10}$ to $\frac{1}{15}$ equals the ratio of 3 to 2.

1. What is the ratio of 5 to 3? of $\frac{1}{5}$ to $\frac{1}{3}$?
2. What is the ratio of 3 to 5? of $\frac{1}{3}$ to $\frac{1}{5}$?
3. What is the ratio of 4 to 5? of $\frac{1}{4}$ to $\frac{1}{5}$?
4. What is the ratio of 5 to 4? of $\frac{1}{5}$ to $\frac{1}{4}$?
5. What is the ratio of 4 to 2? of $\frac{1}{4}$ to $\frac{1}{2}$?
6. What is the ratio of 2 to 4? of $\frac{1}{2}$ to $\frac{1}{4}$?

7. What is the ratio of 6 to 4? of $\frac{1}{6}$ to $\frac{1}{4}$?

8. What is the ratio of 4 to 6? of $\frac{1}{4}$ to $\frac{1}{6}$?

9. What is the ratio of 9 to 6? of $\frac{1}{9}$ to $\frac{1}{6}$?

10. What is the ratio of 6 to 9? of $\frac{1}{6}$ to $\frac{1}{9}$?

11. What is the ratio of 18 to 27? of $\frac{1}{18}$ to $\frac{1}{27}$?

12. What is the ratio of 6 to 18? of $\frac{1}{6}$ to $\frac{1}{18}$?

13. What is the ratio of 7 to 28? of $\frac{1}{7}$ to $\frac{1}{28}$?

14. What is the ratio of 12 to 36? of $\frac{1}{12}$ to $\frac{1}{36}$?

15. What is the ratio of 45 to 9? of $\frac{1}{45}$ to $\frac{1}{9}$?

16. Make similar sentences: The ratio of 5 to 3 equals the ratio of $\frac{1}{8}$ to $\frac{1}{5}$.

17. What is the ratio of 17 to 18? of $\frac{1}{17}$ to $\frac{1}{18}$?

18. What is the ratio of 23 to 37? of $\frac{1}{23}$ to $\frac{1}{37}$?

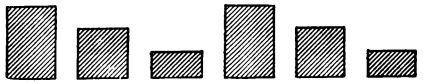
19. What is the ratio of $\frac{1}{8}$ to $\frac{1}{4}$? of $\frac{1}{6}$ to $\frac{1}{8}$? of $\frac{1}{12}$ to $\frac{1}{15}$?

20. What is the ratio of $\frac{1}{3}$ to $\frac{1}{6}$? of $\frac{1}{3}$ to $\frac{1}{12}$? of $\frac{1}{12}$ to $\frac{1}{6}$?

21. What is the ratio of $\frac{1}{27}$ to $\frac{1}{9}$? of $\frac{1}{14}$ to $\frac{1}{7}$? of $\frac{1}{21}$ to $\frac{1}{7}$?

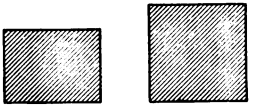
22. Make sentences like these: 2 is the ratio of 8 to 4, of $\frac{1}{4}$ to $\frac{1}{8}$. $\frac{3}{2}$ is the ratio of 27 to 18, of $\frac{1}{18}$ to $\frac{1}{27}$. $\frac{1}{2}$ is the ratio of 17 to 34, of $\frac{1}{34}$ to $\frac{1}{17}$.

23. Make sentences like these: The ratio of 9 to 12 equals the ratio of $\frac{1}{12}$ to $\frac{1}{9}$. The ratio of 45 to 30 equals the ratio of $\frac{1}{30}$ to $\frac{1}{45}$.



Remark. — A unit equal to $\frac{1}{2}$ of 8 and $\frac{1}{3}$ of 12 is the largest exact measure common to 8 and 12. A unit equal to $\frac{1}{2}$ of $\frac{1}{12}$ or $\frac{1}{3}$ of $\frac{1}{8}$ is the largest exact measure common to the units $\frac{1}{12}$ and $\frac{1}{8}$. Hereafter the term *largest measure* will be used for largest exact measure common.

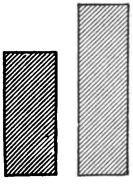
1. $\frac{3}{4}$ is the ratio of a to b. What part of a is the largest measure of a and b? What part of b is the largest measure?



2. If $\frac{2}{5}$ is the ratio of h to k, what part of h is the largest measure of each? Draw h and k.

3. If $\frac{3}{7}$ is the ratio of m to n, what part of m is the largest measure of each?

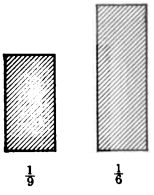
4. $\frac{3}{8}$ is the ratio of $\frac{1}{8}$ to $\frac{1}{2}$. What part of $\frac{1}{8}$ is the largest measure of each?



5. Make many similar sentences: $\frac{4}{7}$ is the ratio of a to b. $\therefore \frac{1}{7}$ of a is the largest measure of a and b. What is the ratio of $\frac{1}{8}$ of a to $\frac{1}{7}$ of b?

6. What is the ratio of $\frac{5}{7}$ to $\frac{5}{8}$? of $\frac{1}{7}$ to $\frac{1}{8}$? What part of $\frac{1}{7}$ is the largest common measure of each? $\frac{1}{5}$ of $\frac{1}{7}$ equals what part of 1?

7. What is the ratio of $\frac{6}{9}$ to $\frac{6}{6}$? of $\frac{1}{9}$ to $\frac{1}{6}$? What part of $\frac{1}{9}$ is an exact measure of $\frac{1}{6}$ and of $\frac{1}{9}$?



If the pupil arrives at the fact that the greatest common measure of $\frac{1}{6}$ and $\frac{1}{9}$ is $\frac{1}{18}$ by a process which he performs according to order, a process which does not bring $\frac{1}{6}$ and $\frac{1}{9}$ into his mental view in such a way that he sees this to be true, is not any conclusion which is based upon this purely *formal for him?* He may find and say that the sum of $\frac{1}{9}$ and $\frac{1}{6} = \frac{5}{18}$, their difference $\frac{1}{18}$, and draw fact after fact, but there is no mental equation, no act which a machine might not perform.

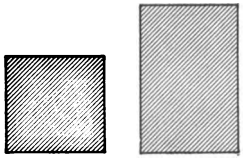
The mind may operate freely and legitimately upon symbols when it has first put meaning into the symbols. The way, and the only way, to put meaning into symbols is by repeated acts of sensing, feeling, and thinking. Through varied experiences ideas grow and language becomes significant.

8. What is the ratio of $\frac{1}{12}$ to $\frac{1}{3}$? What part of $\frac{1}{12}$ is a measure of each? $\frac{1}{3}$ of $\frac{1}{12}$ equals what part of 1? What part of 1 is a measure of $\frac{1}{12}$ and $\frac{1}{3}$? How many of these measures in $\frac{1}{12}$? in $\frac{1}{3}$?

9. Draw the units $\frac{1}{12}$ and $\frac{1}{8}$. What is the part of 1 that is the largest measure of $\frac{1}{12}$ and $\frac{1}{8}$? of $\frac{1}{15}$ and $\frac{1}{10}$? of $\frac{1}{18}$ and $\frac{1}{12}$? of $\frac{1}{2}$ and $\frac{1}{4}$? of $\frac{1}{6}$ and $\frac{1}{3}$? of $\frac{1}{8}$ and $\frac{1}{2}$? of $\frac{1}{18}$ and $\frac{1}{6}$? of $\frac{1}{48}$ and $\frac{1}{24}$? of $\frac{1}{175}$ and $\frac{1}{150}$? of $\frac{1}{7}$ and $\frac{1}{11}$?

10. If x equals $\frac{1}{12}$ of a and y equals $\frac{1}{8}$ of a, what part of a is the largest measure of x and y? What, then, is the part of x that is the largest measure of x and y? x equals what part of a? What, then, is the part of a that is the largest measure?

11. What is the largest measure of $\frac{1}{21}$ and $\frac{1}{14}$?



(*a*) $\frac{1}{21} = \frac{2}{3}$ of $\frac{1}{14}$.

(*b*) $\therefore \frac{1}{21} = \frac{2}{3}$ of $\frac{1}{14}$.

(*c*) $\therefore \frac{1}{2}$ of $\frac{1}{21}$, or $\frac{1}{42}$, is the l. m. of $\frac{1}{21}$ and $\frac{1}{14}$.

12. What is the l. m. of $\frac{1}{10}$ and $\frac{1}{16}$? $\frac{1}{10} = \frac{8}{5}$ of $\frac{1}{16}$. $\therefore \frac{1}{8}$ of $\frac{1}{10}$ is the l. m. of $\frac{1}{10}$ and $\frac{1}{16}$.

1. What is the sum of $\frac{5}{6}$ and $\frac{3}{8}$?

Make sentences similar to these: $\frac{4}{3}$ is the ratio of $\frac{1}{8}$ to $\frac{1}{6}$. $\frac{1}{24}$ is the l. m. $\frac{5}{6} = \frac{20}{24}$. $\frac{3}{8} = \frac{9}{24}$. $\frac{29}{24}$ is the sum of $\frac{5}{6}$ and $\frac{3}{8}$.

What is the sum of —

2. $\frac{5}{7}$ and $\frac{3}{4}$?
3. $\frac{3}{5}$ and $\frac{7}{12}$?
4. $\frac{5}{6}$ and $\frac{7}{11}$?
5. $\frac{5}{8}$ and $\frac{7}{15}$?
6. $\frac{4}{8}$ and $\frac{2}{20}$?
7. $\frac{4}{5}$ and $\frac{7}{15}$?
8. $\frac{3}{4}$ and $\frac{2}{5}$?
9. $\frac{5}{6}$ and $\frac{2}{3}$?
10. $\frac{3}{14}$ and $\frac{4}{21}$?
11. $\frac{5}{11}$ and $\frac{3}{2}$?
12. $\frac{5}{6}$ and $\frac{2}{3}$?
13. $\frac{8}{21}$ and $1\frac{3}{2}$?
14. $\frac{6}{7}$ and $1\frac{3}{2}$?
15. $\frac{2}{3}$ and $\frac{4}{7}$?
16. $\frac{3}{14}$ and $\frac{4}{21}$?
17. $\frac{3}{8}$ and $\frac{5}{36}$?
18. $\frac{3}{8}$ and $\frac{7}{15}$?
19. $\frac{4}{5}$ and $\frac{7}{15}$?
20. $\frac{3}{18}$ and $\frac{5}{36}$?
21. $\frac{2}{27}$ and $\frac{5}{9}$?
22. $\frac{5}{8}$ and $\frac{7}{19}$?

23. What is the sum of $\frac{7}{12}$, $\frac{5}{9}$, $\frac{3}{8}$?

$\frac{42 \cdot 40 \cdot 27}{72} = 1\frac{7}{12}$. What is the l. m. of $\frac{1}{9}$ and $\frac{1}{8}$? of $\frac{1}{72}$ and $\frac{1}{12}$? What, then, is the l. m. of the three units?

$\frac{8}{9}$ is the ratio of $\frac{1}{9}$ to $\frac{1}{8}$.

$\therefore$ $\frac{1}{8}$ of $\frac{1}{9}$, or $\frac{1}{72}$, is the l. m. of the two units. $\frac{1}{72}$ is a measure of $\frac{1}{12}$.

$\therefore$ $\frac{1}{72}$ is the l. m. of the three units.

24. What is the l. m. of $\frac{4}{9}$, $\frac{5}{6}$, $\frac{7}{12}$, $\frac{11}{18}$?

$\frac{1}{18}$ is the l. m. of the 9ths, 6ths, and 18ths.

$\frac{1}{18} = \frac{2}{3}$ of $\frac{1}{12}$.

$\therefore$ $\frac{1}{36}$ is the l. m. of the four units.

25. What is the largest measure of $\frac{3}{7}$, $\frac{2}{21}$, $\frac{4}{5}$, $\frac{5}{14}$, $\frac{18}{42}$?

$\frac{1}{42}$ is the l. m. of 7ths, 21sts, 14ths, and 42ds.

$\frac{1}{42}$ equals $\frac{5}{42}$ of $\frac{1}{5}$.

$\therefore$ $\frac{1}{5}$ of $\frac{1}{42}$, or $\frac{1}{210}$, is the l. m. of the five units.

26. What is the sum of $\frac{3}{7}$, $\frac{2}{21}$, $\frac{4}{5}$, $\frac{18}{42}$?

Practice finding the l. m. in each problem. Practice finding the sums in each:

1. $\frac{11}{24}$, $\frac{5}{12}$, $\frac{3}{8}$.
2. $\frac{8}{15}$, $\frac{7}{12}$, $\frac{2}{5}$.
3. $\frac{7}{15}$, $\frac{5}{9}$, $\frac{3}{5}$.
4. $\frac{13}{30}$, $\frac{7}{10}$, $\frac{9}{25}$.
5. $\frac{5}{16}$, $\frac{11}{20}$, $\frac{3}{5}$.
6. $\frac{2}{9}$, $\frac{7}{18}$, $\frac{3}{7}$, $\frac{8}{21}$.
7. $\frac{1}{2}$, $\frac{4}{7}$, $\frac{5}{8}$, $\frac{7}{15}$.
8. $\frac{5}{9}$, $\frac{7}{12}$, $\frac{5}{6}$, $\frac{11}{20}$.
9. $\frac{7}{10}$, $\frac{7}{15}$, $\frac{11}{18}$, $\frac{3}{5}$
10. $\frac{11}{15}$, $\frac{7}{4}$, $\frac{7}{20}$, $\frac{3}{16}$.
11. $\frac{8}{15}$, $\frac{9}{20}$, $\frac{7}{9}$, $\frac{13}{18}$.
12. $\frac{5}{8}$, $\frac{5}{4}$, $\frac{5}{3}$, $\frac{5}{6}$.
13. $\frac{1}{2}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{7}{10}$, $\frac{5}{12}$.
14. $\frac{7}{8}$, $\frac{4}{9}$, $\frac{5}{16}$, $\frac{7}{18}$.

Find the sum of —

1. $4\frac{1}{2}$ + $6\frac{3}{4}$
2. $3\frac{4}{9}$ + $9\frac{2}{3}$
3. $5\frac{2}{3}$ + $7\frac{1}{2}$
4. $5\frac{3}{8}$ + $7\frac{2}{3}$
5. $8\frac{4}{5}$ + $5\frac{2}{7}$
6. $4\frac{7}{8}$ + $11\frac{5}{8}$

Find the sum of —

1. $4\frac{8}{9}$, $13\frac{4}{9}$, and $36\frac{5}{8}$.
2. $35\frac{2}{3}$, $54\frac{2}{3}$, $36\frac{4}{7}$.
3. $24\frac{8}{9}$, $48\frac{1}{2}$, $27\frac{3}{4}$.
4. $36\frac{4}{7}$, $59\frac{3}{4}$, $32\frac{7}{12}$.
5. $74\frac{8}{15}$, $18\frac{3}{10}$, $60\frac{5}{9}$, $23\frac{9}{18}$.
6. $35\frac{3}{8}$, $13\frac{4}{5}$, $67\frac{9}{12}$, $39\frac{9}{10}$.
7. $17\frac{1}{2}$, $27\frac{3}{8}$, $28\frac{2}{3}$, $15\frac{5}{6}$, $37\frac{3}{4}$.

1. How much greater is $\frac{19}{24}$ than $\frac{7}{18}$?

$\frac{57-28}{72} = \frac{29}{72}$. $\therefore \frac{19}{24}$ is $\frac{29}{72}$ greater than $\frac{7}{18}$.

What is the difference between $\frac{19}{24}$ and $\frac{7}{18}$? What must be added to $\frac{7}{18}$ to make the sum equal $\frac{19}{24}$? If $\frac{7}{18}$ is taken out of $\frac{19}{24}$, what is left? $\frac{19}{24} - \frac{7}{18}$ equals what? What is the ratio of $\frac{19}{24}$ to $\frac{7}{18}$? of $\frac{7}{18}$ to $\frac{19}{24}$?

In each what must be added to the lower unit to make it equal to the upper?

2. $\frac{11}{12}$ / $\frac{5}{6}$
3. $\frac{5}{7}$ / $\frac{3}{5}$
4. $\frac{7}{18}$ / $\frac{1}{24}$
5. $\frac{5}{16}$ / $\frac{7}{32}$
6. $\frac{19}{30}$ / $\frac{9}{21}$
7. $\frac{12}{17}$ / $\frac{3}{51}$

8. What must be added to $13\frac{5}{7}$ to make the sum equal to $24\frac{2}{7}$?

$24\frac{2}{7}$ $\frac{5}{7}$ and what equal $\frac{2}{7}$?
$13\frac{5}{7}$ 14 and what equal 24?
$10\frac{4}{7}$

$\therefore 10\frac{4}{7}$ must be added to $13\frac{5}{7}$ to make the sum equal to $24\frac{2}{7}$.

$13\frac{5}{7}$ is how much less than $24\frac{2}{7}$? $24\frac{2}{7}$ is how much greater than $13\frac{5}{7}$? What is the difference between $24\frac{2}{7}$ and $13\frac{5}{7}$? If $13\frac{5}{7}$ is taken out of $24\frac{2}{7}$, what remains? What is the sum of $10\frac{4}{7}$ and $13\frac{5}{7}$?

9. What must be added to $24\frac{8}{9}$ to make the sum equal to $73\frac{1}{5}$?

$73\frac{9}{45}$ $\frac{40}{45}$ and what equal $\frac{54}{45}$?
$24\frac{40}{45}$ 25 and what equal 73?
$48\frac{14}{45}$

$73\frac{1}{3}$ is how much more than $24\frac{8}{9}$? $73\frac{1}{3}$ less $24\frac{8}{9}$ equals what? What is the difference between $73\frac{1}{3}$ and $24\frac{8}{9}$? What is the sum of $48\frac{14}{15}$ and $24\frac{8}{9}$?

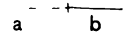
10. $23\frac{5}{8} - \frac{7}{8} = ?$
11. $47\frac{1}{8} - \frac{3}{4} = ?$
12. $98\frac{6}{7} - 15\frac{10}{11} = ?$
13. $13\frac{11}{14} - 9\frac{6}{7} = ?$
14. $68\frac{1}{9} - 345\frac{10}{21} = ?$
15. $75\frac{3}{17} - 38\frac{1}{11} = ?$
16. $32\frac{11}{20} - 18\frac{3}{15} = ?$
17. $68\frac{2}{9} - 35\frac{10}{21} = ?$

How much greater is —

1. $83\frac{5}{8}$ than $69\frac{5}{7}$?
2. $64\frac{1}{8}$ than $35\frac{3}{4}$?
3. $28\frac{7}{15}$ than $13\frac{7}{10}$?
4. $356\frac{2}{3}$ than $178\frac{1}{12}$?
5. $125\frac{3}{5}$ than $77\frac{5}{8}$?
6. $323\frac{1}{2}$ than $84\frac{4}{5}$?

1. Draw a line and separate it so that the ratio of the parts is 1.

2. Fold a paper so that the ratio of the larger to the smaller part is 2. What is the ratio of the smaller to the larger part? What is the ratio of each part to the entire paper? of the paper to each part?

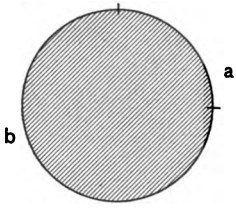
3. What is the ratio of a to b? of b to a? 

of a to the sum of a and b? of b to the sum? of the sum to b? of the sum to a? Think these ratios again and again.

4. Show parts of the blackboard that have the ratio 2. What is the ratio of each of these parts to the entire board? of the board to each part?

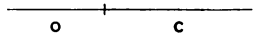
5. Observe a and b. Tell all the ratios that you see. The unit has been separated so that the ratio of the parts is what? Give the ratio of a and b; of a to the sum; of b to the sum; of the sum to a, and of the sum to b. What ratios have you given? Do not observe, but image the units and think all the ratios again.

6. Show me the circumference of the circle. Show me the arc b; the arc a. What is the ratio of arc a to arc b? of b to a? of b to the circumference? of a to the circumference? of the circumference to each arc?

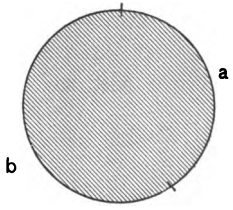


7. Draw a rectangle. Separate the rectangle into two parts so that the ratio of the rectangle to the larger part is $\frac{4}{3}$.

8. What are the ratios of the parts of this line? What other ratios do you see? Think these 6 ratios again and again.

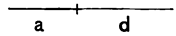


9. Draw a staff and separate it so that the staff equals $\frac{5}{2}$ of the shorter part. What is the ratio of the staff to the longer part? If you break a stick so that the shorter part equals $\frac{3}{5}$ of the longer, the longer equals what part of the entire stick?



10. The arc a is 2 and the arc b is 3. What ratios do you find? What is the ratio of each arc to the circumference? of the circumference to each arc?

11. Measure each part of this line by $\frac{1}{3}$ of a. Discover all the ratios that you can. Think the ratios without observing the lines.



12. If a farm is divided between a son and a daughter so that the daughter's portion equals $\frac{4}{5}$ of the son's, what part of the farm does each receive?

13. Draw a line 12 in. long. Divide it so that one part shall equal $\frac{1}{3}$ of the other. What is the length of each part?

14. A and B mow 12 acres of grass. A mows $\frac{1}{3}$ as much as B. How much does each mow?

15. A hat and cloak cost \$12. The cost of the hat equals $\frac{1}{3}$ of the cost of the coat. What is the cost of each?

16. When the time past noon equals $\frac{1}{3}$ of the time to midnight, what is the time? Make *one* drawing which will illustrate each of the last *four* problems.

1. Divide the blackboard so that $\frac{1}{2}$ of one part equals $\frac{1}{3}$ of the other.

2. Divide the blackboard so that $\frac{1}{2}$ of one part is as large as $\frac{2}{3}$ of the other. What ratios do you see?

3. If $\frac{1}{2}$ of the sugar in a equals $\frac{1}{3}$ of the sugar in b, what is the ratio of the sugar in a to the sugar in b? What is the ratio of the sugar in each to the sugar in both? of the sugar in both to the sugar in each?

4. If $\frac{2}{3}$ of the distance you walk in the forenoon equals $\frac{3}{4}$ of the distance you walk in the afternoon, which distance is the greater, the forenoon or the afternoon distance? Draw a line to represent $\frac{2}{3}$ of the forenoon distance. Draw a line to represent $\frac{3}{4}$ of the afternoon distance. Complete each distance. Which is the greater? What is true of the length of these lines?

5. If $\frac{2}{7}$ of a equals $\frac{2}{5}$ of b, which is the larger unit?

6. If $\frac{5}{7}$ of a equals $\frac{3}{4}$ of b, which is the greater?

7. If $\frac{1}{3}$ of x equals $\frac{1}{7}$ of y, which is the greater?

8. If $\frac{2}{3}$ of A's money equal $\frac{3}{4}$ of B's, and together they have \$40, how much has each?

9. Divide a line 30 in. long into two parts so that their *relative* length shall be the same as that of $\frac{1}{2}$ of an inch and $\frac{2}{3}$ of an inch.

10. Divide the unit 48 into two parts so that their relative size shall be the same as that of $\frac{2}{3}$ and $\frac{3}{7}$.

11. $\frac{3}{4}$ of a equals $\frac{5}{8}$ of b. Give ratio of b to a.

1. Draw the face of the clock.

2. How many hours from noon until midnight?

3. When it is 1 P.M. what is the time past noon? What is the time to midnight?

4. When it is 1 P.M. the time past noon equals what part of the time to midnight?

5. When it is 1 P.M. what is the ratio of the time from noon until midnight to the time past noon?

6. When it is 1 P.M. what is the ratio of the time from noon until midnight to the time to midnight?

7. When the time past noon equals $\frac{1}{11}$ of the time to midnight, what is the ratio of the time to midnight to 12 hr.?

8. When it is 2 P.M. the time past noon equals what part of the time to midnight?

9. When it is 2 P.M. what is the ratio of 12 hr. to the time to midnight?

10. What is the time when the time to midnight equals $\frac{1}{3}$ of the time past noon?

11. It is 3 P.M. Tell all the ratios that you can.

12. What is the time when the time to midnight equals 3 times the time past noon?

13. Suppose it is 4 P.M. Tell the ratio of the time past noon to the time to midnight; of the time to midnight to the time past noon; of the time past noon to the time from noon until midnight; of the time from noon until midnight to the time past noon; of the time from noon until midnight to the time to midnight. Tell all these ratios again.

14. It is 5 P.M. Tell all the ratios that you can.

15. What time is it when the time past noon equals the time to midnight?

16. It is 7 P.M. Tell all the ratios that you can.

17. It is 10 P.M. Tell all the ratios you can.

1. If the area of a circle equals the area of a square, and the area of a triangle equals the area of a square, what is the ratio of the circle to the triangle? Why?

2. Pupils show that things which are equal to the same thing are equal to each other.[1]

If the unit a equals $\frac{2}{3}$ of the unit b, and the unit c equals the unit b, what is the ratio of a to c?

3. Give five questions similar to the above.

4. What is the relation of a to $\frac{2}{3}$ of a? If $\frac{2}{3}$ of a equals b, what is the relation of a to b? If a equals $\frac{3}{2}$ of $\frac{2}{3}$ of a, why does a equal $\frac{2}{3}$ of b?

5. What is the relation of x to $\frac{3}{4}$ of x? If $\frac{3}{4}$ of x equals y, what is the relation of x to y? If x equals $\frac{4}{3}$ of $\frac{3}{4}$ of x, why does x equal $\frac{4}{3}$ of y?

6. What is the relation of c to $\frac{5}{7}$ of c? If $\frac{5}{7}$ of c equals d, what is the relation of c to d? Why?

7. What is the relation of m to $\frac{7}{6}$ of m? If $\frac{7}{6}$ of m equals o, what is the relation of m to o? Why?

8. What is the relation of a to $1\frac{1}{2}$ a's? If $1\frac{1}{2}$ a's equal b, what is the relation of a to b?

9. Make and answer many questions similar to the following: If $1\frac{2}{7}$ a's equal b, what is the ratio of a to b? a equals $\frac{7}{9}$ of $1\frac{2}{7}$ a's; then a equals what part of b?

10. If $\frac{3}{4}$ of a equals $\frac{1}{2}$ of b, what is the relation of a to $\frac{1}{2}$ of b? Why? What is the ratio of a to b?

[1] I have found many high school pupils demonstrating (?) propositions in geometry without any real apprehension of the equality of ratios involved when they assert that "Two things each equal to a third are equal to each other." Intellectual haziness can be cleared only by contact with realities. Such contact is the basis of inferences concerning things beyond perception.

"Most of the difficulties in this science are difficulties rather of intuition than of reasoning." — LEWES, *Problems of Life and Mind*, Vol. I, p. 388.

11. If $\frac{3}{4}$ of a equals $\frac{3}{7}$ of b, what is the relation of a to $\frac{3}{7}$ of b? of a to b? Show objectively.

12. If $\frac{5}{4}$ of a equals $\frac{3}{7}$ of b, what is the relation of a to $\frac{3}{7}$ of b? What is the ratio of a to b?

13. If $1\frac{1}{4}$ a's equal $\frac{3}{7}$ of b, what is the relation of a to $\frac{3}{7}$ of b?

To Teacher. — In solving the following problems have pupils represent things in given relations. Then review without object, and give similar problems. Real progress is shown by growing power to think of things not present to sense.

1. If $\frac{4}{7}$ of A's capital equals $\frac{2}{3}$ of B's, why does the ratio of A's capital to B's equal $\frac{4}{7}$ of $\frac{2}{3}$, or $\frac{4}{7}$ of B's?

2. Two men start from opposite points 12 miles apart and walk toward each other until they meet. $\frac{2}{3}$ of the distance A walks equals $\frac{2}{5}$ of the distance B walks. How many miles does each walk?

3. When $\frac{2}{3}$ of the time past midnight equals $\frac{2}{5}$ of the time to noon, what is the hour of day?

4. A pole, the length of which was 120 ft., was in the air and water. $\frac{2}{3}$ of the length of the part in the water equaled $\frac{3}{4}$ of the length of the part in the air. What was the length of the part in the air?

5. When $\frac{2}{3}$ of the time past noon equals $\frac{3}{4}$ of the time to midnight, what is the hour of day?

6. A horse and carriage cost \$198, and $\frac{1}{2}$ of the cost of the carriage equaled $\frac{2}{5}$ of the cost of the horse. What was the cost of the horse?

7. A and B build 198 rods of fence in a certain time. If $\frac{1}{2}$ of the work A does equals $\frac{2}{5}$ of the work B does, how many rods does each build?

8. If you know the cost of a watch and chain and the part of the cost of the watch to which the cost of the chain is equal, how would you find the cost of each?

9. If the cost of a watch and chain were $\$x$, and the cost of the chain were equal to $\frac{1}{6}$ of the cost of the watch, what part of $\$x$ would each cost? If the cost of the chain were equal to $\frac{2}{7}$ of the cost of the watch, what part of $\$x$ would each cost?

10. A watch and chain cost \$70, and $\frac{2}{7}$ of the cost of the watch equaled $\frac{3}{8}$ of the cost of the chain. What was the cost of each?

11. John and James saw a pile of wood in x hours. If James does $\frac{2}{3}$ as much as John, what part of the work does each do?

12. A and B can do a piece of work in 10 days. If A does $\frac{2}{3}$ as much as B, what part of the work does each do?

13. How does the entire amount of work compare with the part each does in 10 days? How long would it take each alone to do the work?

14. If A does $\frac{3}{5}$ as much work as B, and both together earn \$32, what is the share of each?

15. Two men enter into a partnership, one investing $1\frac{3}{5}$ times as much as the other. If they make \$640, what is each man's share?

16. If the time past noon equals $1\frac{3}{5}$ times the time to midnight, the time past noon equals what part of 12 hours? What is the hour of the day?

17. A stick was broken into two pieces so that $\frac{3}{5}$ of the longer piece equaled the shorter; the difference in the length of the two pieces was 4 in. What was the length of the whole stick?

18. A hat cost $\frac{2}{3}$ as much as a cloak, and the difference in their cost was \$8. What was the cost of each?

19. John and James buy a sled. John pays $\frac{3}{4}$ as much as James, and James pays \$.12 more than John. How much does the sled cost?

1. Represent 1 yd. of carpet 1 yd. wide. Represent 1 yd. of carpet $\frac{3}{4}$ of a yard wide. What is the ratio of the former to the latter? of the latter to the former?

2. What is the ratio of a floor covered by 20 yd. of carpet $\frac{3}{4}$ of a yard wide to a floor covered by 20 yd. of carpet 1 yd. wide?

3. If 60 yd. of carpet will cover a floor equal to $\frac{5}{8}$ of a floor to be covered, the floor to be covered equals what part of the floor covered by 60 yd.?

4. What is the ratio of a floor covered by x yd. of carpet 1 yd. wide to a floor covered by x yd. $\frac{7}{8}$ of a yard wide?

5. What is the ratio of a floor covered by x yd. of carpet 1 yd. wide to a floor covered by x yd. $1\frac{1}{4}$ yd. wide?

6. What is the ratio of 70 yd. of carpet 1 yd. wide to 70 yd. $\frac{7}{8}$ of a yard wide?

7. If it takes 70 yd. of carpet 1 yd. wide to cover a floor, how many yards $\frac{3}{4}$ of a yard wide will it take to cover the same floor? 70 yd. $\frac{3}{4}$ of a yard wide will cover what part of the floor? What is the ratio of the floor to $\frac{3}{4}$ of it? What, then, is the ratio of the required number of yards to 70 yd.?

8. If 10 yd. of carpet 5 ft. wide will cover a floor, how many yards 7 ft. wide will cover the floor?

9. If a floor contains 30 sq. yd., what part of 30 yd. of carpet $\frac{5}{4}$ yd. wide will be required to cover it?

10. If a floor contains 25 sq. yd., what part of 25 yd. of carpet $\frac{5}{3}$ yd. wide will be required to cover it?

11. How many yards of carpet $\frac{4}{3}$ yd. wide will be required to carpet a room 18 ft. long and 15 ft. wide?

12. How many yards of carpet $\frac{5}{3}$ yd. wide will carpet a room 24 ft. long and 23 ft. wide?

13. Write 5 examples similar to the 12th, and state in one sentence what equals the number of yards required in each.

1. At \$2 a bushel, how many bushels of apples can be bought for \$2? At \$4 a bushel, what can be bought for \$2? Then what is the ratio of the quantity of apples which can be bought for any sum at \$4 a bushel to the quantity which can be bought for an equal sum at \$2 a bushel?

2. How does the amount of flour I can purchase for \$$x$ when flour is \$6 a barrel compare with the amount I can purchase for \$$x$ when flour is \$12 a barrel? For \$6 how much flour can I purchase at \$6 a barrel? For \$6 how much flour can I purchase at \$12 a barrel? What, then, is the ratio of the flour that can be bought for any sum at \$6 a barrel to the flour that can be bought for an equal sum at \$12 a barrel? If for \$$x$, at \$6 a barrel, 5 barrels can be bought, how much can be bought for \$$x$ at \$12 a barrel?

3. What is the ratio of the number of cases of fruit which can be bought for \$$x$ at \$15 each to the number of cases which can be bought for \$$x$ at \$5 each?

4. Write 3 problems similar to the 3d, and state what comparisons you make in solving them.

5. What is the ratio of the flour which a baker can afford to put into a 6¢ loaf when flour is \$3 a barrel, to the flour he can put into it when flour is \$5 a barrel?

6. If a 10¢ loaf weighs 30 oz. when flour is \$5 a barrel, what ought it to weigh when flour is \$10 a barrel?

7. If a 5¢ loaf weighs 20 oz. when flour is \$3½, what ought it to weigh when flour is \$7 a barrel?

8. If, when flour is \$6 a barrel, you can buy a 28-oz. loaf for 6¢, what ought a 6¢ loaf to weigh when flour is \$4 a barrel?

9. What is the ratio of the number of dozen oranges which can be bought for \$$x$ at \$$\frac{3}{7}$ a dozen to the number that can be bought for \$$x$ at \$$\frac{3}{8}$ a dozen?

1. What is the cost of 5 pk. of gooseberries at $12\frac{1}{2}¢$ a quart?

2. At $75¢$ a quart, what is the cost of $1\frac{1}{2}$ gal. of alcohol?

3. If $2\frac{1}{2}$ lb. of butter cost $50¢$, 1 lb. costs how many 5ths of $50¢$?

4. If $5\frac{1}{2}$ bu. of beans cost $3.30, what does 1 bu. cost?

5. If John can hoe a garden in 1 week, working 6 hr. a day, in what part of a week can he hoe it working 9 hr. a day? Why?

6. If $12\frac{1}{2}$ lb. of sugar cost $x¢$, what do $37\frac{1}{2}$ lb. cost?

7. If 7 boys can mow a tennis ground in 10 min., how long does it take 3 boys? Why?

8. If x yd. of carpet $1\frac{1}{2}$ yd. wide will cover a floor, how many yards $\frac{3}{4}$ of a yard wide will cover the same floor?

9. Carrie can make a dress in 6 days and Fannie in 9 days; if they both work together 3 days, what part of the work is done?

10. John can mow a lawn in 6 hr. and Clarence in 4 hr. If each work 2 hr., what part of the lot is mowed?

11. Frank, Charles, and Henry saw $\frac{1}{8}$ of a pile of wood in a day, and Frank and Charles saw $\frac{1}{12}$ of the pile in a day. What part of the pile does Henry saw in a day?

12. If Julia does 3 times as much as Jessie, and they together do $\frac{1}{6}$ of a piece of work in a day, in what time can each do the work?

13. Jane makes an apron in $\frac{2}{3}$ of a day, and Lucy makes one in $\frac{3}{4}$ of a day. How many aprons can both make in a day?

14. If Leroy can wash a carriage in 4 hr., and with the help of Harry can do it in 2 hr., what part of the work does each do?

15. James and Robert do a piece of work in x hr.; James does twice as much as Robert. What is the part of the work done by each? Represent by drawing.

16. A man and boy saw a pile of wood in 7 hr., the man sawing twice as much as the boy. What part of the wood does the boy saw? In what time could he saw it all?

17. Mr. Brown and his son fence a lot in 8 hr., the son doing half as much as the father. In what time could the father do the work alone?

18. Draw a rectangle and separate it into 12 equal parts. Think of it as a flower bed which Clara, Olive, and Mary can weed in 2 hr.; Clara and Olive in 4 hr.; and Clara and Mary in 3 hr. Show the part of the flower bed Olive and Mary can weed in 1 hr. In what time can Olive and Mary weed the bed?

19. A can do a piece of work in 4 days and B in 3 days. What is the ratio of the work that they can both do in 4 days to the work that A can do in 4 days? What is the ratio of the work that both can do in 3 days to the work that B can do in 3 days?

20. 32 miles equals $\frac{1}{8}$ of the distance between two places. 32 miles and what equals $\frac{1}{6}$ of the distance?

21. If $\$x$ equals the cost of $\frac{5}{8}$ of a lot, what equals the cost of $\frac{3}{4}$ of a lot?

22. If $1\frac{6}{4}$ of a barrel of flour costs $\$2\frac{1}{2}$, what does $2\frac{6}{8}$ of a barrel cost?

23. A tank has two pipes, one filling it at the rate of 70 gal. an hour, the other emptying it at the rate of 45 gal. an hour. How many gallons of water will there be in the tank at the end of 6 hr.?

24. A man walks 18 mi. while a boy walks 13. At this rate how many miles does the man walk while the boy walks 65 mi.?

25. A boy lost $\frac{3}{4}$ of his marbles and then bought $\frac{1}{2}$ as many as he had left, when he had 12 marbles. How many had he at first?

26. The difference between $\frac{2}{3}$ and $\frac{3}{4}$ of the length of a line is 2 in. What is the length of the line?

27. The difference between $\frac{4}{5}$ and $\frac{5}{6}$ of a surface is 7 sq. in. How many square inches in the area of the surface?

28. What unit is as much greater than $\frac{3}{4}$ as $\frac{2}{3}$ is less?

29. If to a blackboard you add an amount equal to its $\frac{1}{2}$, $\frac{1}{3}$, and 30 sq. ft. the sum equals three times the number of square feet in the blackboard, what is its area?

30. $\frac{3}{5}$ of a quart equals what part of 3 qt.? of 3 gal.?

31. $\frac{5}{6}$ of a peck equals what part of 2 bu.? Review.

32. What is the ratio of 1 lb. of butter to 4 oz.? to 1 oz.? to $\frac{1}{4}$ oz.?

33. $\frac{1}{2}$ pt. equals what part of $\frac{1}{2}$ gal.? Review.

34. What is the ratio of 4 yd. to $\frac{2}{3}$ of a foot?

35. $\frac{1}{7}$ of 1 equals what part of 5?

36. $\frac{1}{100}$ of 1 equals what part of 5? of 4? of $\frac{1}{2}$?

37. $\frac{1}{100}$ of 1 equals what part of $\frac{1}{4}$? of $\frac{1}{3}$? of $\frac{1}{8}$?

38. 6 equals how many 4ths of 8?

39. 5 equals how many 4ths of 7?

40. 6 equals how many hundredths of 5? of 4? of $\frac{1}{2}$? of $\frac{3}{4}$?

41. The ratio of the number of cents Howard has to the number Lawrence has is $\frac{3}{4}$. The sum of their money is 77¢. How much has each?

42. Two boys bought a melon for 15¢. One paid 6¢ and the other 9¢. What part of the melon should each have?

43. A man has $2200. The ratio of the amount he has in gold to the amount in notes is $\frac{5}{6}$. How many dollars in gold has he? How many in notes?

44. A and B are in partnership; their profits are \$400. A's investment equals $1\frac{3}{5}$ times B's. What is each man's share of the profits?

45. Find the cost of 3000 cedar posts at $\$2\frac{1}{2}$ per hundred.

46. What is the rate per minute of a train which runs 80 mi. in 1 hr. and 40 min.?

47. If a man earns $\frac{5}{8}$ of a dollar in $\frac{3}{5}$ of a day, how much does he earn in $\frac{3}{4}$ of a day? What ratios are equal?

48. If $\frac{3}{4}$ of an article is sold for a sum equal to what $\frac{7}{8}$ of it cost, what is the ratio of the gain to the cost? Show by drawing.

49. A barrel of beef which costs \$12.50 is retailed at $8\frac{1}{2}¢$ a pound. The gain equals what part of the cost?

50. Out of $176\frac{3}{15}$ tons of coal, how many families can be supplied with $\frac{1}{5}$ of a ton each?

51. What number diminished by $\frac{1}{2}$ of $\frac{3}{7}$ of itself leaves a remainder of 275?

52. Two prisms have equal bases. What is the ratio of the larger to the smaller, if one is $3\frac{1}{2}''$ high and the other is $2''$ high?

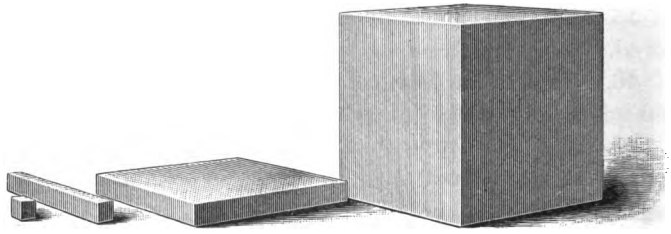
53. Draw a square equal to $\frac{1}{2}$ of a square $4''$ long. What is the ratio of the square A to the square B, if the length of A equals the distance from the center of B to one of its corners?

54. How many tin boxes $4''$ in each dimension can be put into a box $13''$ wide, $8''$ high, and $15''$ long?

55. How many prisms 2 by 5 by 7 can be put into a box 4 by 10 by 14? into a box 6 by 15 by 21?

56. How many spheres $2''$ in diameter can be put into a tin box $6''$ by $4''$ by $8''$? into a box $6''$ by $4''$ by $2''$? into a box $6''$ by $2''$ by $2''$?

Place solids representing 1, 10, 100, 1000 so that the pupils can handle and observe them. See pp. 189–191, "Elementary Arithmetic," for method of work.



1. If a is 1, what is d? If b is 1, what is a? If c is 1, what is b?

2. If c is 1, what is each of the others?

3. If b is 1, what is each of the others?

4. If d is 1, what is each? If d is 2, what is each?

5. If d is $\frac{1}{2}$, what is each? If d is $\frac{1}{3}$, what is each? If d is 5?

6. Associate the following names with a variety of units having the ratios 1000, 100, 10, .1, .01, .001:

1	1000	500	250	8000	2000
.1	100	50	25	800	200
.01	10	5	$2\frac{1}{2}$	80	20
.001	1	$\frac{1}{2}$	$\frac{1}{4}$	8	2

7. If d is .5, what is each of the other units?

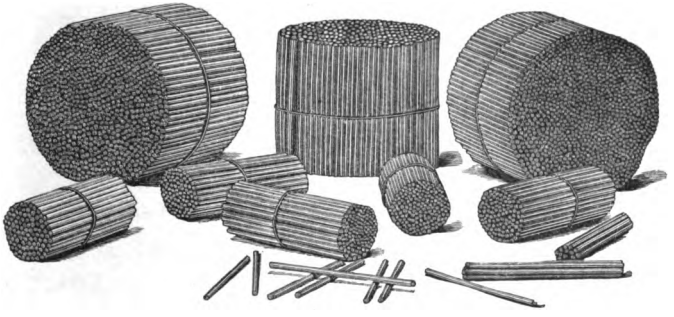
8. If c is 4000, what is each of the others? if it is 9000?

9. If b is 7000, what is each of the other units?

10. If c is $\frac{2}{3}$, what does each of the other units equal?

11. What is the ratio of $\frac{1}{2}$ of b to $\frac{1}{2}$ of each of the others? of $\frac{1}{2}$ of a? of $\frac{1}{2}$ of d? of $\frac{1}{4}$ of d to $\frac{1}{4}$ of each of the others?

Place bundles of a thousand sticks, several bundles of hundreds, and tens, and a number of ones where they can be handled.



1. If we call one of the largest bundles 1, what shall we call the next in size? the next? What shall we call one of the sticks?

2. Show a 1. Show .1. Show .01. Show .001.

3. Show .3. Show .02. Show .004. Show .324.

Give much practice in each of the following exercises:

(*a*) Pupils select numbers of tenths, hundredths, and thousandths, and tell what they have. *Ex.* Pupil selects .1, .03, .002, and says, "I have .1, .03, .002, or 132 thousandths." Teacher writes .132.

(*b*) Write on blackboard and have pupils find units named. *Ex.* Write .025. Pupil finds .02 and .005.

(*c*) Pupils read the following and find units named: .02, .5, .04, .040, .23, .230, .1, .10, 1.2, .02, .002, .2, .02, .020, .3, .30, .300, .1111.

(*d*) Teacher or pupil dictates and pupils write. *Ex.* Write 3 tenths; 24 tenths; 84 tenths; 5 hundredths; 27 hundredths; 240 hundredths; 785 hundredths; 2 thousandths; 456 thousandths; 400 thousandths; 2400 thousandths.

Remark. — If, after the above study, a pupil cannot write decimals, review the exercises. Do not explain, but keep their attention on the decimal relations. The pupil will grow into these forms of expression as into others. Here, as elsewhere, the language should be freely given, but not made the object of thought.

Tell the ratio of —

1.	1 to .1.	1 to .01.	1 to .001.
2.	.1 to .01.	.01 to .1.	2 to .2.
3.	.5 to 5.	.20 to .2.	.3 to .003.
4.	.003 to .3.	.7 to .007.	.001 to .1.
5.	.100 to .1.	.01 to .001.	.04 to .004.

1. How many tenths in 1? in 2? in $1\frac{1}{2}$? in $2\frac{1}{2}$?
2. How many hundredths in .1? in .2? in $.1\frac{1}{2}$? (Read in $1\frac{1}{2}$ tenths.) in $.2\frac{1}{2}$?
3. How many hundredths in $.0\frac{1}{2}$? in $.01\frac{1}{2}$? in $.0\frac{1}{3}$? in .04?
4. How many thousandths in 1? in .1? in .01? in $.01\frac{1}{2}$?
5. How many thousandths in .01? in .06? in $.05\frac{1}{2}$?
6. How many tenths in 1? in $1\frac{1}{3}$? in $1\frac{2}{3}$?
7. How many hundredths in 3? in 3.1? in 3.7?
8. How many thousandths in 4? in $4\frac{1}{2}$? in 4.5?
9. Review and write answers to each.

Each of the following equals how many thousandths?

1.	.1.	4.2.	2.
2.	.7.	.5.	.04.
3.	.3.	6.42.	3.7.
4.	.04.	24.35.	.26.

Each of the following equals how many ten thousandths?

1.	.8.	4.5.	.04.
2.	.85.	9.	4.
3.	.164.	33.75.	.295.

1. $\frac{3}{4}$ is the ratio of what to 24? What is the simplest expression of the ratio of 18 to 24?

2. $\frac{15}{21}$ is the ratio of what to 27? What is the simplest expression of the ratio of 15 to 27?

3. .15 is the ratio of what to 100? .15, or $\frac{15}{100} = \frac{3}{20}$. What is the ratio of 3 to 20? What, then, is the ratio of 15 to 100?

Express the following ratios in the simplest form —

4.	.75.	3.5.	37.25.	27.4000.
5.	.125.	8.94.	.875.	28.0004.
6.	.225.	4.75.	.375.	.075.
7.	.35.	6.25.	.625.	28.3.

1. What is $\frac{3}{5}$ of 19?

What is $\frac{1}{5}$ of 15?

3.8 The 4 equals how many tenths?

$\frac{3 \quad 19}{5} = 11.4$. $\frac{1}{5}$ of 4.0 equals what?

What, then, is $\frac{1}{5}$ of 19?

What, then, equals $\frac{3}{5}$ of 19?

2. What is $\frac{1}{3}$ of 7, expressed by 3 decimal places?

$\frac{1}{3}$ of 6 equals what?

$\frac{1}{3}$ of .9 equals what?

$\frac{1}{3} = 2.333\frac{1}{3}$. $\frac{1}{3}$ of .09 equals what?

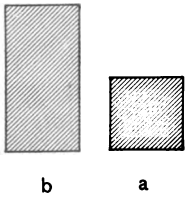
$\frac{1}{3}$ of .010 equals what?

What, then, is $\frac{1}{3}$ of 7?

3. If a is 1, what is b? What is the relation of $\frac{1}{3}$ of 2 to $\frac{1}{3}$ of 1? Then $\frac{1}{3}$ of 2 equals how many $\frac{1}{3}$ of 1? Then $\frac{2}{3}$ of 1 equals $\frac{1}{3}$ of what?

What is the relation of $\frac{1}{4}$ of 3 to $\frac{1}{4}$ of 1? Then $\frac{1}{4}$ of 3 equals how many $\frac{1}{4}$ of 1? Then $\frac{3}{4}$ of 1 equals $\frac{1}{4}$ of what?

$\frac{2}{5}$ of 12 equals $\frac{1}{5}$ of what? $\frac{1}{5}$ of 3 · 12's equals $\frac{3}{5}$ of what? What is the ratio of $\frac{3}{5}$ of 12 to $\frac{1}{5}$ of 3 · 12's?



$\frac{3}{8}$ of 7 equals $\frac{1}{8}$ of how many 7's? $\frac{3}{4}$ of 100% equals $\frac{1}{4}$ of how many 100%? $\frac{3}{4}$ of 100 apples equals $\frac{1}{4}$ of how many 100 apples?

4. Review pp. 245, 246, "Elementary Arithmetic."
5. What is the ratio of $\frac{3}{5}$ of 1 to $\frac{1}{5}$ of 3?
6. What is the ratio of $\frac{1}{7}$ of 5 to $\frac{5}{7}$ of 1?
7. Which weighs the more, $\frac{4}{5}$ of 1 lb. or $\frac{1}{5}$ of 4 lb.?
8. What is $\frac{5}{5}$ of 1 or $\frac{1}{5}$ of 5?

$\frac{1}{8}$ of 4.8 equals what?

.625

8 ⟌ 5.000

$\frac{1}{8}$ of .16 equals what?

$\frac{1}{8}$ of .040 equals what?

What, then, is $\frac{1}{8}$ of 5 or $\frac{5}{8}$ of 1?

9. Express $5\frac{3}{4}$ decimally.

3 equals how many tenths?

$\frac{1}{4}$ of 2.8 equals what?

$\frac{1}{4}$ of .20 equals what?

$5\frac{3}{4} = 5.75$.

What, then, is $\frac{1}{4}$ of 3, or $\frac{3}{4}$ of 1, expressed decimally?

What, then, is $5\frac{3}{4}$ expressed decimally?

Express each of the following decimally. Do not carry the expression to more than 3 decimal places.

10. $2\frac{1}{2}$.	14. $\frac{11}{14}$.	18. $12\frac{1}{2}$.	22. $\frac{7}{8}$.
11. $\frac{7}{3}$.	15. $6\frac{2}{3}$.	19. $7\frac{5}{8}$.	23. $\frac{13}{15}$.
12. $6\frac{1}{8}$.	16. $5\frac{5}{8}$.	20. $\frac{1}{212}$.	24. $16\frac{3}{8}$.
13. $4\frac{3}{4}$.	17. $\frac{11}{21}$.	21. $37\frac{1}{2}$.	25. $62\frac{1}{2}$.

State the ratio of a to b and of b to a.

	a	b	a	b
1.	100.	25.	100.	$16\frac{2}{3}$.
2.	100.	75.	100.	$33\frac{1}{3}$.
3.	100.	20.	100.	$66\frac{2}{3}$.
4.	100.	40.	100.	$83\frac{1}{3}$.
5.	100.	60.	100.	$12\frac{1}{2}$.

1. What is the sum of $25 \cdot 84$?

$4)\overline{8400}$ What is the sum of $100 \cdot 84$'s?

$\overline{2100}$. What, then, is the sum of $25 \cdot 84$'s?

Find the sum of —

2. $25 \cdot 648$.
3. $75 \cdot 9.85$.
4. $33\frac{1}{3} \cdot 8.45$.
5. $66\frac{2}{3} \cdot 75$.
6. $60 \cdot 78.45$.
7. $80 \cdot 64$.
8. $16\frac{2}{3} \cdot 425.79$.
9. $83\frac{1}{3} \cdot 947$.
10. $12\frac{1}{2} \cdot 57.88$.
11. $37\frac{1}{2} \cdot 77.45$.
12. $87\frac{1}{2} \cdot 77.45$.
13. $250 \cdot 7853$.

1. At $\$33\frac{1}{3}$ an acre, what is the cost of 9.47 acres of land?

$315.66\frac{2}{3}$ At $\$100$ an acre, what does the land cost?

$3)\overline{947}$. What, then, is the cost at $\$33\frac{1}{3}$ an acre?

2. What is the cost of 9.47 acres of land at $\$25$ an acre? at $\$37\frac{1}{2}$?

3. At $\$1$ a box a grocer paid $\$200$ for fruit. What would it have cost at $\$.75$ a box? at $\$.60$?

4. At $\$1$ a bushel, what is the cost of 160 bushels of flax? at $\$.87\frac{1}{2}$ a bushel?

5. What is the ratio of the cost of government land at $\$1.83\frac{1}{3}$ an acre to the cost at $\$1$ an acre?

6. If m equals $.66\frac{2}{3}$ of the cost of an article, what equals the cost?

7. What is the ratio of a rectangle to .8 of it?

8. There are 37 sq. in. in .9 of a rectangle. How many square inches in the rectangle?

9. If .7 of the distance between two places equals 168 miles, what is the distance? Show by drawing.

10. 2400 miles equals .7 of the distance between two places. What is the distance? Show by drawing.

11. A merchant buys caps for \$1 and sells for \$1.62½. The gain equals what part of the cost?

12. .12½ of the money a man invested equaled his profit. If \$600 was the profit, what equaled the amount invested?

13. A man bought a horse for \$$x$ and sold it for \$180. The selling price equaled ⅘ of \$$x$. What was the cost of the horse?

14. What part of the cost of hats, at \$1 each, must be added to the cost to find the cost at \$1.33⅓ each? at \$1.37½ each? at \$1.87½ each? at \$1.62½ each?

15. \$160 is the cost of apples at \$1 a bushel. If the same number of bushels cost the sum of \$160 and ⅛ of \$160, what is the price per bushel?

16. Carpeting which cost \$1 a yard is sold for 87½¢ a yard. The loss equals what part of the cost?

17. A furniture dealer paid \$100 for chairs at \$1 each. He sold them for ⅕ more than they cost him. What was the selling price of each chair? What did he receive for all?

18. A milliner bought hats at \$1 and marked them to sell for ¼ more than they cost. What was the marked price?

She sold them for ⅓ less than the marked price. What did she receive for them? Show by drawing. Did she make or lose by buying and selling the hats?

19. A merchant bought dress goods at 75¢ a yard, and marked it to sell at 20% profit, but sold it at 10% below marked price. Did he make or lose by the transaction?

20. At 40¢ a pound, what is the cost of 5 lb. 12 oz. of butter?

21. What is the cost of 4 pk. and 7 qt. of cherries at \$1 a peck?

22. At \$2 a bushel, what is the cost of 3 pk. of plums?

1. If 10 weighs 18, what does 17 weigh?

What are 1.7 ($1\frac{7}{10}$) of 18?

What is .1 of 18? *Ans.* 1.8 (eighteen tenths).

What, then, are 1.7 of 18? $17 \cdot 1.8 = 30.6$.

2. What is .25 of .75?

$\frac{.75}{4} = ?$ What is a simpler expression than $\frac{75}{100}$?

What is $\frac{1}{4}$ of .75?

3. Review pp. 243, 244, "Elementary Arithmetic."

4. What is 2.48 ($2\frac{48}{100}$) of 2.75?

$\frac{62 \quad .11}{\cancel{248} \cdot \cancel{2.75}} = 6.82.$ What is $\frac{1}{25}$ of 2.75?

$\cancel{100}$ What is $\frac{1}{4}$ of 248?

$\cancel{4}$ What, then, is $\frac{1}{100}$ of $248 \cdot 2.75$?

5. .25 of .36?
6. .5 of 68?
7. 3.2 of 70?
8. .75 of 2.05?
9. .39 of .875?
10. .04 of 6.5?
11. .025 of 823?
12. .3 of 750?
13. 1.2 of 3.12?
14. 24.6 of 98 ft.?
15. 2.3 of 6.25 mo.?
16. .27 of $98.75?
17. $\frac{3}{5}$ of $5.75?
18. $\frac{9}{4}$ of 74.56?

1. What is the cost of 52.75 bu. of oats at 35¢ a bushel?

$\frac{2.11}{35 \cdot \cancel{52.75}} = 18.46\frac{1}{4}$. At $1 a bushel, what is the cost of the oats?

$\cancel{100}$ What, then, is the cost at 35¢?

4

2. What is the cost of 16.5 yd. of muslin at 8¢ a yard?
3. What does a man earn in 15.5 days at $2.65 per day?
4. What is the cost of 12,200 shingles at $4.80 per 1000?
5. What is the cost of 10,240 bricks at $5.25 per 1000?
6. What is the cost of 650.75 bu. of flax at $1.46 per bushel?

7. What is the cost of 2600 envelopes at \$2.16 per 100?

8. At 65¢ per pound, what must be paid for 254.75 lb. of tea?

9. At \$5.35 per ton, what must be paid for 12.5 tons of coal?

10. What is the cost of 14.5 yd. of calico at $6\frac{1}{4}$¢ a yard?

11. A man sold 79.25 cords of wood at \$3.08 a cord. How much did he receive?

12. What is the cost of 675 books at \$1.12$\frac{1}{2}$ each?

1. What is .0$\frac{3}{4}$ of 824 lb.? (*Read:* What is $\frac{3}{4}$ of .1 of 824 lb.?)

20.6 What is .1 of 824?

$\frac{3 \cdot 82.4}{4} = 61.8$. What, then, is .0$\frac{3}{4}$?

2. What is .00$\frac{3}{4}$ of 824 lb.?

3. What is .0$\frac{3}{7}$ of \$630?

4. What is .0$\frac{3}{5}$ of 84$\frac{5}{10}$?

5. What is .0$\frac{3}{8}$ of .24?

6. What is .2$\frac{1}{4}$ of 96?

2.4 What is .1 of 96?

$\frac{9 \cdot 9.6}{4} = 21.6$. What, then, is .0$\frac{3}{4}$ of 96?

7. What is .2$\frac{1}{3}$ of 56.7?

8. What is .2$\frac{3}{5}$ of 8.49?

9. If a barrel of apples costs \$5.75, what is the cost of 5.3 bbl.?

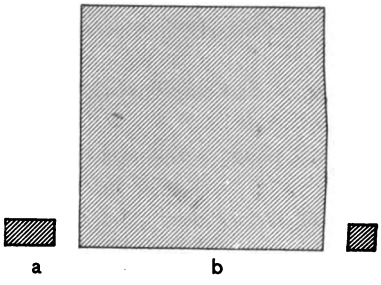
10. What is the value of .2$\frac{1}{3}$ acres of land at \$37.50?

11. If .05 of a farm is worth \$472, what is the farm worth?

12. If 824 lb. is .0$\frac{3}{4}$ of the wheat in a bin, what is the amount of wheat in the bin?

13. $\frac{2}{3}$ of x equals $\frac{1}{3}$ of what?

What part of x equals $\frac{1}{3}$ of $2x$?



1. If .02 is the ratio of a to b, what is the ratio of b to a?

What is the ratio of a to the sum of a and b? of b to the sum? of the sum to a? of the sum to b? of a to their difference? of b to their difference?

2. What is the ratio of .07 to 3.5?

$\frac{7}{350} = \frac{1}{50} = .02.$

3.5 equals how many hundredths?

What is the ratio of 7 hundredths to 350 hundredths?

What, then, is the ratio of .07 to 3.5?

3. If .02 is the ratio of .07 to 3.5, what is the ratio of 3.5 to .07?

4. What is the ratio of 3.5 to .07?

$7)\overline{350}\,50 = 50.$

What is the ratio of 350 hundredths to 7 hundredths?

What, then, is the ratio of 3.5 to .07?

Why does the ratio of 350 hundredths to 7 hundredths equal the ratio of 3.5 to .07?

5. What is the ratio of 12.5 to .45?

$\frac{1250}{45} = \frac{250}{9} = 27.777.$

What is the ratio of 1250 hundredths to 45 hundredths?

What, then, is the ratio of 12.5 to .45?

Why does the ratio of 1250 to 45 equal the ratio of 250 to 9?

6. What is the ratio of .45 to 12.5?

$$\begin{array}{r} .036 \\ 1250)\overline{45.00} \\ 3750 \\ \hline 7500 \\ 7500 \end{array}$$

What is the ratio of 45 hundredths to 1250 hundredths?

What, then, is the ratio of .45 to 12.5?

What is the ratio of a to b and of b to a?

a	b	a	b
.95.	1.9.	6.	42.
.08.	.4.	52.5.	.015.
8.12.	4.	.0256.	.27.
.25.	7.5.	3.	.4.
1.5.	1.05.	24.	.0001.
682.5.	25.	\$16.	\$.06¼.
.015.	.5.	\$12.62½.	\$5.
682.5.	25.	\$16.	\$.06¼.
45.825.	150.	\$5000.	\$.125.
.01.	4.	\$75.	\$1.25.
15.77.	.083.	\$67.83.	\$.75.
3.56.	3.9.	\$.875.	\$.12½.
6.25.	.05.	135.05.	.327.
625.	.05.	17.28.	1728.
786.	5.	135.	.37½.
.05.	64.5.	\$22.10.	1.70.
625.	.08.	1.70.	22.10.

In each of the above, express the ratio of a to b and b to a in hundredths, thus:

$\frac{95}{190} = \frac{1}{2} = .50.$ .50 is the ratio of .95 to 1.9 expressed in hundredths.

$\frac{190}{95} = 2 = 2.00.$ 2.00 is the ratio of 1.9 to .95 expressed in hundredths.

1. If two shovels are worth $1.70, how many can be bought for $22.10?

2. If 26 shovels cost $22.10, how many can be bought for $1.70?

3. If a dozen roses can be bought for $87\frac{1}{2}¢$, how many dozen can be bought for $10.50?

4. At $2.50 per 100, how many 100 bananas can be bought for $40?

5. If you buy bananas at $2.50 per 100, and sell them for $20¢$ more than cost, what do you receive for the bananas?

6. If oranges are $35¢$ a dozen, how many can be bought for $10.70?

7. At $1.75 per yard, how many yards of silk can be bought for $18.20?

8. If a half dozen dress patterns, each containing 7 yd., sell for $68.25, what is that per yard?

What is the ratio of $2\frac{1}{2}$ to $3\frac{1}{4}$ expressed in hundredths?

$\frac{10}{13}$ is the ratio of $2\frac{1}{2}$ to $3\frac{1}{4}$ expressed as a common fraction. $\frac{10}{13} = .76\frac{12}{13}$.

Express the relation of a to b in hundredths:

a	b	a	b
7.	$2\frac{1}{2}$.	$2\frac{1}{4}$.	$3\frac{1}{2}$.
$\frac{1}{4}$.	$\frac{1}{2}$.	$\frac{1}{2}$.	$\frac{4}{5}$.
$37\frac{1}{2}$.	5.	$1\frac{1}{8}$.	$2\frac{1}{3}$.
$5\frac{1}{2}$.	$6\frac{1}{2}$.	$\frac{1}{2}$.	$1\frac{3}{2}$.
5.	$5\frac{1}{2}$.	$\frac{1}{2}$.	$\frac{3}{5}$.
.75.	.6.	6.22.	5.6.

Express in hundredths:

$\frac{10}{13}$, $\frac{3}{20}$, $\frac{7}{30}$, $\frac{2}{5}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{1}{6}$, $\frac{5}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{12}$, $\frac{5}{12}$, $\frac{7}{12}$, $\frac{1}{16}$, $\frac{3}{16}$.

1. 7 is the ratio of what unit to \$9? What is the ratio of \$63 to \$9? \$9 equals what part of \$63?

2. $\frac{3}{5}$ is the ratio of what unit to 45? What is the ratio of 45 to 27? 27 equals what part of 45? $1\frac{2}{3}$ is the ratio of what unit to 27?

3. .07 is the ratio of what unit to \$600? What is the ratio of $7 \cdot$ \$6 to \$600? $7 \cdot$ \$6 equals how many hundredths of \$600? \$600 equals how many 7ths of $7 \cdot$ \$6?

What is the ratio of the wheat that can be bought for \$600 to the wheat that can be bought for $7 \cdot$ \$6?

The wheat that can be bought for $7 \cdot$ \$6 equals what part of the wheat that can be bought for \$600?

4. What is $.00\frac{1}{2}$ of \$8400? (*Read:* What is $\frac{1}{2}$ of $\frac{1}{100}$ of \$8400?) What is .01 of \$8400? What, then, is $.00\frac{1}{2}$ of \$8400?

5. What equals $.00\frac{3}{4}$ of 754 lb.? (What is $\frac{3}{4}$ of $\frac{1}{100}$ of 754 lb.? What is .01 of 754 lb.? What, then, is $.00\frac{3}{4}$ of 754 lb.?

6. What equals $.02\frac{2}{5}$ of \$24.56?

1.885

$$\frac{3 \cdot \cancel{7}.\cancel{5}\cancel{4}}{\cancel{4}} = 5.655.$$

1. What is the ratio of $.03\frac{1}{2}$ to .01?

Ans. $\frac{7}{2}$ is the ratio of $.03\frac{1}{2}$ to .01.

2. State the ratio of each of the following to .01: $.03\frac{2}{3}$, $.02\frac{1}{5}$, $.03\frac{3}{4}$, $.09\frac{1}{2}$, $.17\frac{1}{8}$, $.10\frac{2}{3}$, $.06\frac{2}{3}$.

3. What is the ratio of .01 to $.03\frac{1}{2}$?

Ans. $\frac{2}{7}$ is the ratio of .01 to $.03\frac{1}{2}$.

4. State the ratio of .01 to each of the following: $.03\frac{2}{3}$, $.02\frac{1}{5}$, $.03\frac{3}{4}$, $.09\frac{1}{2}$, $.17\frac{1}{8}$, $.07\frac{2}{3}$, $.06\frac{2}{3}$, $.04\frac{5}{8}$, $.00\frac{3}{4}$, $.00\frac{7}{8}$, $.00\frac{8}{8}$, $.004\frac{4}{5}$.

5. What is the ratio of .01 to each of the following?

$.7$, $.2\frac{1}{4}$, $.9\frac{2}{3}$, $.007$, $.33\frac{1}{3}$, 3.7.

1. What is .00¾ of $824?

$\frac{3 \cdot 8.24}{4} = ?$

What is .01 of $824?
What, then, is .00¾ of $824?

2. What is .00⅞ of $2114?
3. What is .00⅞ of $840?
4. What is .00⅜ of 947 ft.?
5. What is .00¾ of 875 da.?
6. What is .02½ of 78 tons?

$\frac{5 \cdot .78}{2} = ?$

What is .01 of 78 tons?
What, then, are .02½ of 78 tons?

7. What is .02⅛ of $729?
8. What is .07¼ of 16.72 bu.?
9. What is .13⅜ of 5764 oz.?

1. What is the ratio of a unit to .07 of the unit?

$\frac{100 \cdot 255}{7} = ?$

$255 is .07 of what unit?
What equals .01 of the unit?
What, then, equals ⅛⅛ of it?

2. 96¢ equals .3 of what?
3. 24 ft. equals .25 of what?
4. 645 pk. equals .60 of what?
5. 231 gal. equals .03½ of what?

$\frac{100 \cdot 2 \cdot 231}{7} = ?$

What is the ratio of .01 to .03½?
.01 of the unit equals what part of 231 gal.?
The unit equals what?

6. $846 are .01½ of what unit?
7. 854.37 are .5⅛ of what unit?
8. 247 yd. are .02⅜ of what?
9. $675.25 are .02¾ of what?
10. $723.54 are .00¾ of what?
11. 68 ft. are .00⅜ of what?
12. 2745 are .00⅝ of what?
13. 32.3 are .0¾ of what?

1. When a ball is thrown into the air, what force pulls it to the earth?

2. Name things upon which this force acts.

3. What can you name that is not influenced by the force of gravity?

4. When a boy slips and falls, what pulls him to the floor?

5. What holds the oceans, lakes, air, etc., in their places?

6. When a dredging boat is unloaded, what pulls the mud, sand, and stones through the water?

7. What pulls rain and snow through the air to the surface of the earth?

8. If you place a stick at the bottom of a vessel of water, what force pushes it to the surface of the water?

9. If a boy uses a pole to raise a window, is the boy or the stick the primary cause of the movement of the window?

10. What force pushes an ascending balloon away from the surface of the earth?

11. If a 6-in. cube of wood weighs as much as a 6-in. cube of water, will the 6-in. cube of wood, if placed in a tub of water, sink to the bottom? Why? If the cube of wood is placed at the bottom of the tub, will it rise?

12. Why does a stone fall through water? Why does a snowflake fall through the air?

13. What is true of the pulling power of gravity upon a stone and upon an equal volume of water? upon a snowflake and an equal volume of air?

14. What causes some soap-bubbles to rise? What is in a soap-bubble?

What is the difference between the air in a soap-bubble and the air about it?

15. What is weight?

16. Which is heavier, a floating balloon or a descending snowflake?

17. Throw a ball into the air. While rising, does it have weight?

18. In which is an apple the heaviest, in water, in air, or in a vacuum?

Specific Gravity. — The specific gravity of any substance is the ratio of the weight of the substance to the weight of an equal volume of water.

The following is the specific gravity of a few liquids and solids:

Ice	.93.	Alcohol	.791.
Iron (wrought)	7.6 to 7.8.	Quicksilver	13.596.
Silver	10.5.	Milk	1.032.
Gold	19. to 19.6.	Anthracite coal	1.8.
Tin	7.29.	Bituminous "	1.25.
Oak	.84.	Lead	11.35.

1. How many ounces in a cubic foot of water if it weighs 62½ lb.?

2. What part of 1000 oz. does a cubic foot of ice weigh? Why? How many pounds does a cubic foot of ice weigh?

3. What is the weight of a 6-in. cube of silver?

4. What is the weight of a gold brick 2 in. by 4 in. by 8 in.?

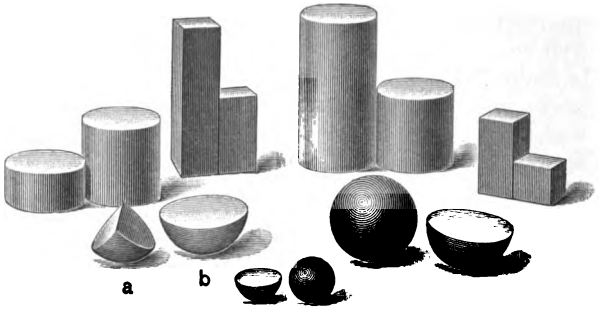
5. If a cubic foot of water weighs 1000 oz. and a cubic foot of quartz 2650 oz., what is the specific gravity of the quartz?

6. How many cubic inches in 75 oz. of alcohol?

7. What is the specific gravity of a bar of iron 4 in. square and 10 in. long, if it weighs 40 lb.?

8. If a substance heavier than water is immersed in water, the water buoys it up just the amount of the weight of the water the substance displaces. If a cubic foot of stone weighs 1500 oz. in water, what is the specific gravity of the stone?

9. If just ½ of a log of wood floats above water, what is its specific gravity?



1. What ratios do you see?

2. If b is 100, what is a? If b is 100% (per cent), what is a?

3. Make similar sentences: If 100% is 6, 50% is 3.

4. 3 equals what % of 6%? $\frac{1}{8}$ equals what % of $\frac{2}{3}$?

5. If a is 50%, what is the sum of a and b?

6. If 150% is 12, what is 100%? 50%?

7. If 150% is 15, what is 100%?

8. 10 equals what % of 10? 6 equals what % of 6?

9. a equals what % of b?

10. $\frac{1}{2}$ of b equals what % of a? of b?

11. What is the ratio of 100% to 50%? of 50% to 100%? What part of 100% equals 50%? If b is 100%, the sum of a and b equals what %?

12. The difference between a and b equals what % of b? of a?

13. If b is 25%, what is a? If b is 10%, what is a? If a is $\frac{1}{2}$%, what is b?

14. This is 50%. Draw a rectangle equal to 100%.

15. This line is 100%. Draw a line equal to 50%; to 200%; to 150%.

16. Observe things in the room and write five statements similar to the following:[1] 2 windows equals 50% of 4 windows. 3 rows of desks equals 50% of 6 rows. $\frac{1}{2}$ the blackboard equals 50% of the entire blackboard.

17. Show by drawings the relative magnitude of things whose ratio equals that of 50% to 100%; of 50% to the sum of 50% and 100%.

18. Write five statements similar to the following: 50% of 8 books equals 4 books. 50% of the coal in the bin equals $\frac{1}{2}$ of it. 50% of 10 hooks equals 5 hooks.

19. 50% is the ratio of what to 4? to 6? to $\frac{1}{2}$? to 20? to $\frac{1}{8}$?

20. Write five questions similar to the following: A boy had 12 marbles and lost 50% of them. How many had he left?

21. What is 50% of each of the following units? 4 pk., 8 ft., \$20, 16 cu. in., 180 mi., 1800 yr., \$$\frac{1}{2}$, 75¢.

22. Give quickly the units of which the following is 50%: 5 lb., 12 bu., \$120, $\frac{2}{3}$, 60 ft., 180 da., 25¢, 5 dimes, $3\frac{1}{2}$, 13 halves, $\frac{1}{8}$, \$150, 75¢.

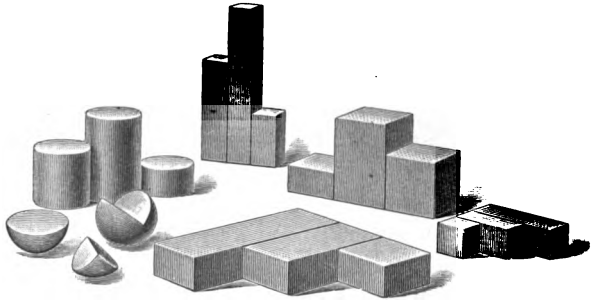
23. 5 lb. equals what % of 10 lb.? $\frac{1}{7}$ equals what % of $\frac{2}{7}$? $\frac{7}{4}$ equals what % of $3\frac{1}{2}$?

24. The ratio of 50% to 100% equals the ratio of 75¢ to what?

25. Express the ratios of a to b and b to a thus: 6 qt. equals 50% of 12 qt.

a	b	a	b
8 qt.	2 gal.	$\frac{4}{5}$.	$\frac{8}{5}$.
25¢.	\$$\frac{1}{2}$.	$2\frac{2}{3}$.	$5\frac{1}{3}$.
15 ft.	10 yd.	18 sq. ft.	12 sq. ft.
2 cu. yd.	54 cu. ft.	1.	$\frac{2}{3}$.

[1] The form equals is used because one whole is to be thought of in relation to another whole.



1. What ratios do you see?

2. If a is 100, what is b? what is c?

3. If a is 100%, what is b? what is c?

4. How many $33\frac{1}{3}\%$ in each unit?

5. What is the ratio of 100% to $33\frac{1}{3}\%$? 3 is the ratio of 100% to what part of $66\frac{2}{3}\%$?

6. What is the ratio of $66\frac{2}{3}\%$ to $33\frac{1}{3}\%$? to $\frac{2}{3}$ of 100%?

7. Find solids, surfaces, and lines respectively having the ratios 100%, $66\frac{2}{3}\%$, and $33\frac{1}{3}\%$, and compare.

8. Observe things in the room and write five sentences similar to the following: 1 box of chalk equals $33\frac{1}{3}\%$ of 3 boxes. 2 desks equals $33\frac{1}{3}\%$ of 6 desks. $\frac{1}{3}$ of the floor equals $33\frac{1}{3}\%$ of it.

9. Recall units and write statements like this: 5 bu. equals $33\frac{1}{3}\%$ of 15 bu. \$25 equals $33\frac{1}{3}\%$ of \$75. $\frac{1}{3}$ of an acre equals $33\frac{1}{3}\%$ of an acre.

If there has been activity of mind through the senses in the previous work; if *varied* experiences have brought simple, basic relations before the pupil under *different*[1] forms, progress will be

[1] "Only after there have been received many experiences which differ in their kinds but present some relation in common, can the first step be taken towards the perception of a truth higher in generality than these different experiences themselves." — HERBERT SPENCER.

easy and rapid at this stage. Let the language employed be given meaning by being used in direct[1] connection with things. To be obliged to translate what should be familiar expressions, instead of at once realizing them in thought, is a waste of energy.

10. $33\frac{1}{3}\%$ is the ratio of each of these units to what? 12 pk., \$6, 400 bu., 90 oz., 35¢, 18 houses, 9 miles.

11. Think quickly $\frac{1}{3}$ of each of the following: \$24, 15¢, 72 lb., \$150, 300 sheep, 1800 books, 42 sq. ft., 1 cu. yd. Think $33\frac{1}{3}\%$ of each of the above units.

12. Observe things in the room and write five sentences like these: $66\frac{2}{3}\%$ is the ratio of 2 hats to 3 hats. $66\frac{2}{3}\%$ is the ratio of 6 slates to 9 slates. $66\frac{2}{3}\%$ is the ratio of $\frac{2}{3}$ of the west wall to the entire wall.

13. What is $66\frac{2}{3}\%$ of five quantities that you observe in the room? *Ex.* $66\frac{2}{3}\%$ of 6 windows is 4 windows.

14. What is $66\frac{2}{3}\%$ of \$18? of 600 pt.? of 1200 bu.?

15. \$12 equals what % of \$18? 400 pt. equals what % of 600 pt.? $66\frac{2}{3}\%$ is the ratio of what to 18?

16. What is the ratio of \$12 to \$18 expressed in %? Name quantities in the room having the ratio of $66\frac{2}{3}\%$. *Ex.* $66\frac{2}{3}\%$ is the ratio of 4 windows to 6 windows.

17. If a is 3%, what is b? c? If a is 1%, what is b? c? If a is $\frac{3}{4}\%$, what is b? c? If b is $\frac{1}{3}\%$, what is c? a? If b is $x\%$, what is c? a?

18. 2 pt. equals what % of 3 qt.? 3 pk. of $1\frac{1}{2}$ bu.? \$1.50 of \$1.00? 3 sq. yd. of 3 yards square? 33 ft. of 11 yd.? 5 ft. of 5 yd.? x ft. of x yd.?

[1] "Words cannot attain definiteness save as living outgrowths of realities, as the exact expression of the phenomena of life." — Dr. Maudsley.

Place prisms having the ratios of the units 1, 2, 3, and 4 where they can be observed.

1. What ratios do you see?

2. If a is 100, what is each of the other units? If a is 100%, what is each of the other units?

3. Tell the ratios of these different %'s.

4. What is the ratio of 25% to 50%? to 75%? to 100%?

5. What is the ratio of each unit to each of the others expressed in %?

6. Make similar sentences: $33\frac{1}{3}\%$ is the ratio of d to the sum of d and c.

7. Introduce exercises similar to those in the preceding lessons to familiarize the ratios of the above %'s.

8. Make problems.

Make similar sentences: 1 is the ratio of 75% to $\frac{3}{4}$ of 100%; to $\frac{3}{2}$ of 50%; to 3 times 25%. 2 is the ratio of 50% to $\frac{1}{4}$ of 100%.

9. Call c 10 and name the others. Give the per cent relation of 10 to 5; to 15; to 20.

10. If a is 4%, what is each of the others? If a is 1%, what is each of the others? If d is $\frac{1}{2}\%$, what is each of the others?

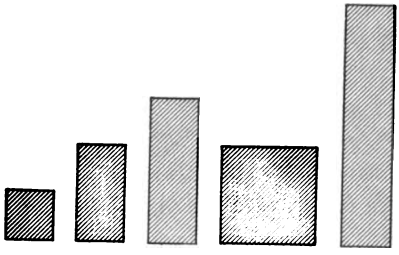
11. Of what are the following 25%? $\frac{4}{5}$, 35 pencils, \$1.25, $2\frac{2}{7}$, $1\frac{1}{3}$.

12. 35¢ equals what % of \$1.40?

What is the ratio of 13 ft. to 26 ft.? of 27 da. to 108 da.? of 3 wk. to 28 da.? of $\frac{2}{5}$ to $1\frac{3}{5}$? of $1\frac{1}{5}$ to $\frac{4}{5}$? of $\frac{1}{2}$ in. square to 1 in. square? of $\frac{x}{2}$ to $2x$? In each what % is the latter of the former?

Review pp. 286–294 inclusive, "Elementary Arithmetic."

Draw rectangles on the blackboard having the ratios of the units 1, 2, 3, 4, 5.



1. What ratios do you see?

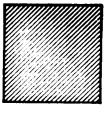
2. If d is 100%, what is each of the other units?

3. Compare each with each of the other four.

4. If e is 100%, what is each of the other units?

5. Compare each with each of the others. Compare without observing the units. If e is 1%, what is each of the others? If e is 5%?

6. Draw the rectangles of which this is 50%; 25%; 75%; 20%; 60%; 40%; 80%; $33\frac{1}{3}$%; $66\frac{2}{3}$%.



7. The ratio of 25% to 75% equals the ratio of $33\frac{1}{3}$% to what? Show by drawing.

8. The ratio of a farm to $66\frac{2}{3}$% of the farm equals the ratio of what part of the farm to 40% of it? Show by drawing.

9. If 100% is \$30, what is 40%? 80%? 20%?

10. If 75¢ is 100%, what is 60%? 80%? 40%? 20%?

11. If 185 bu. is 100%, what is 60%? 80%? 20%?

12. If 80% is one ton, 20% equals how many lb.? 80%?

13. If 60% is 1 sq. yd., 100% is how many sq. ft.?

14. How much is 40% of 40 acres? 100% of 40 acres? 40% of \$250? of 1 sq. in.? of $2\frac{1}{2}$ sec.? of $\frac{1}{2}$ lb.? of $\frac{3}{4}$ pt.?

15. x equals the cost of a book which sells for 20% above cost. Draw figures having the ratio of the selling

price to the cost. If the cost of a book is $\$\frac{1}{2}$, what is the selling price? $\frac{3}{5}$ equals how many 5ths of $\frac{1}{2}$? $\frac{3}{5}$ equals what % of $\frac{1}{2}$? The difference between $\frac{3}{5}$ and $\frac{1}{2}$ equals what % of $\frac{1}{2}$?

16. At 20% gain my profit is 5¢. What is the cost, or 100%? What is the selling price?

17. If 75% of an acre equals 120 sq. rd., 50% of an acre equals how many sq. rd.?

18. x equals 50% of what? $66\frac{2}{3}$% of what?

19. $\frac{1}{4}$ of a blackboard equals what % of $\frac{1}{2}$ of it?

20. A 2-in. square equals what % of a 4-in. square?

21. The perimeter of a 2-in. square equals what % of the perimeter of a 4-in. square?

22. The perimeter of a rectangle, 1×2, equals what % of the perimeter of a 2-in. square?

23. The perimeter of a 1-in. square equals what % of the perimeter of a 2-in. square?

24. 60% of 40 in. equals how many ft.?

25. A farmer had 150 sheep. He sold 40% of them. How many had he left?

26. A coat cost $15, and a vest $66\frac{2}{3}$% as much. What was the cost of both?

27. The area of a 1-in. square equals what % of the area of a 2-in. square?

28. 40% of a section of land equals 256 acres. How many acres in the section?

29. 1200 lb. equals 60% of a ton. How many lb. in the ton?

30. If x equals 80%, what equals 100%?

31. What is the ratio of the gain to the cost if a dealer buys hats for $4 and sells them for $5? if he buys for $5 and sells for $6? if he buys for $3 and sells for $4? if he buys for 50¢ and sells for $1.00?

32. A boy bought a knife for 80¢ and sold it at a profit

of 20%; for how much did he sell it? What is the ratio of the cost to the selling price? of the selling price to the cost?

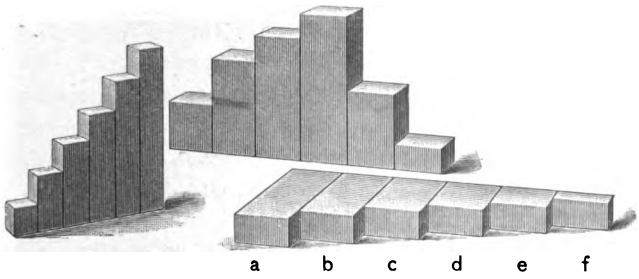
33. If the ratio of $40 to the cost of a horse is 20%, what is the cost of the horse?

34. If in a district of 2500 persons the number of children attending school is 500, what % of the population attends school?

35. If B's property is 100% and A's is 40% more, what % is A's? What is the ratio of B's property to A's? Of A's to B's?

36. John sold a sled for $16, which was 60% more than the cost. What was the ratio of the selling price to the cost? of the cost to the selling price? What was the cost? If he had sold it for 60% less than the cost, for how much would he have sold it?

Place prisms having the ratios 1, 2, 3, 4, 5, 6 where they can be observed.



1. If *a* equals 100%, what does each of the others equal?

2. Make sentences similar to these: The sum of $16\frac{2}{3}$% and $33\frac{1}{3}$% equals 50%. $83\frac{1}{3}$% is $16\frac{2}{3}$% less than 100%.

3. Each unit equals what % of each of the other units? *e* equals 200% of *f*, $66\frac{2}{3}$ of *d*, etc.

4. Introduce a variety of objects in which the above relations may be discovered.

5. 22 is what %, if 132 is 100%? 22 equals what % of 132? of 88?

6. 40 is what %, if 48 is 100%? 40 equals what % of 48?

7. Find $66\frac{2}{3}\%$ of 5 yd.; 8 ft.; 1 cu. yd. Find $16\frac{2}{3}\%$; $83\frac{1}{3}\%$.

8. Make and answer similar problems: When $83\frac{1}{3}\%$ is 15, what is 100%? 15 equals $83\frac{1}{3}\%$ of what? 12 equals how many eighths of 48? 80 equals how many hundredths of 800? 4 equals what % of 24?

9. What is the ratio of $33\frac{1}{3}\%$ to 50%?

When 50% is 96, what is $33\frac{1}{3}\%$?

10. 50% of the weight of a bbl. of flour is 98 lb. $33\frac{1}{3}\%$ of the barrel equals what?

11. The cost of a book is 40¢ and the cost of a tablet 8¢. The cost of the tablet equals what % of the cost of the book?

12. John had 72¢. He gave $83\frac{1}{3}\%$ of his money to a blind man. How much had he left?

What is the ratio of the amount left to the amount given?

13. A 2-ft. square equals what % of 2 sq. ft.?

14. The surface of a 2-in. cube equals what % of the surface of a 4-in. cube?

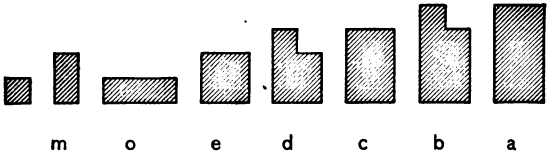
15. The top of a table equals 300% of the top of a desk; if the table is 3 ft. square, what is the area of the top of the desk?

16. 40 sq. rd. equals what % of an acre?

17. A rectangle 80 rods wide and 160 rods long equals what % of a square mile?

18. $\frac{1}{8}$ equals what % of $\frac{1}{2}$? of $\frac{5}{6}$? $\frac{1}{2}$ equals what % of $\frac{1}{3}$? of $\frac{2}{3}$?

Draw rectangles on the blackboard having the ratios of the units 1, 2, 3, 4, 5, 6, 7, 8.



1.[1] If *a* equals 100%, what does each of the other units equal?

2. Add by 12½% to 100%.

3. Call *a* x and name the others.

4. What is the ratio of 100% to 50%? to 25%? to 12½%? to 62½%? to 75%? to 87½%?

What is the ratio of 50% to 25%? to 12½%? to 62½%? to 87½%?

5. 12½% equals what part of 100%? 37½% of a unit equals what part of 100% of it?

6. What is the ratio of 62½% to 12½%? What part of the unit is 62½% of it?

7. Pupils draw on blackboard various figures having these ratios.

Select figures in different groups and have pupils give ratios to other units.

8. If 12½% is taken out of a unit, what % of the unit remains?

9. The sum of 75% and 12½% of a unit equals what % of the unit?

10. Draw the rectangle of which this □ equals 37½%; 87½%; 75%.

11. ⅜ equals how many thirds of ⅜? 100% equals how

[1] "The knowledge of a color by its name can only be taught through the eye. . . . Technical terms must be associated *immediately* with the perception to which they belong." — Dr. Whewell.

many thirds of $37\frac{1}{2}\%$? $\frac{8}{8}$ equals how many thirds of $37\frac{1}{2}\%$?

12. If 9 pk. is $37\frac{1}{2}\%$ of a unit, why may we infer that the unit equals $\frac{8}{3}$ of 9 pk.?

13.[1] Make similar sentences: If x equals $62\frac{1}{2}\%$ of m, $\frac{8x}{5} = m$.

14. Of what unit is each of the following $37\frac{1}{2}\%$? 6 pk.; 12 pk.; 15 pk.; 270 pk.; $\frac{6}{5}$ pk.; $\frac{3}{5}$ pk.; $\frac{2}{4}$ pk.

15. 10 bu. is $62\frac{1}{2}\%$ of how many bu.? What part of the unit is given? What did you compare? The ratio of the unit to $62\frac{1}{2}\%$ of the unit equals the ratio of what to 10 bu.?

16. Answer each of the next three questions and tell what ratios are equal:

21 yd. is $87\frac{1}{2}\%$ of how many yd.?
\$300 is $83\frac{1}{3}\%$ of how many dollars?
x bu. is $37\frac{1}{2}\%$ of how many bu.?

17. Write sentences similar to the following: 6 apples are $37\frac{1}{2}\%$ of 8·2 apples.

18. Think of a rectangle $1'' \times 5''$ and call it 5. Tell how many % less than 5 are 4, 3, 2, and 1, respectively.

19. Assume 6 equal objects to be 100%. Tell how many % less than 6 are 5, 4, 3, 2, and 1, respectively. Tell how many % more than 6 are 7, 8, 9, 10, 11, 12, and 13, respectively.

20. What is $12\frac{1}{2}\%$ of 640 acres? How many sq. rd. in $12\frac{1}{2}\%$ of 1 acre? 320 rd. equals what % of 1 mi.? How many rd. in $12\frac{1}{2}$ % of 1 mi.?

21. 32 equals what % of 64? of 256? of 128? of 48?

22. 63 is what % of 72? of 84? of 126?

[1] "The growth of simple reasoning involves representation of perceptions; and the growth of complex reasoning involves representation of the results of simple reasoning." — HERBERT SPENCER.

23. 63 is what % when 168 is 100% ?

24. Sold 40 sheep of a flock of 64; what % was left? What % was sold ?

What is the ratio of the number sold to the number left ?

25. Make similar sentences: If $37\frac{1}{2}\%$ is 45 ft., $62\frac{1}{2}\%$ $= \frac{5 \cdot 45}{3}$ ft.

To Teacher. — Permit no attempt to solve problems which are not in the mind.[1] Until a question is entertained, it cannot be answered.

Through the ease and correctness of the pupil's expression both by hand and by language, decide whether the work is adapted to his ability. The pupil who finds difficulty in seeing $62\frac{1}{2}\%$ as $\frac{5}{3}$ of $37\frac{1}{2}\%$ is not ready to make the statement called for in Prob. 25. "Explaining" in such work tends to clog the mind. The pupil sees or he does not see. The preparation for complex problems is the gradual development of more complex faculty. We promote such development by exercise in right directions. The average pupil whose powers have been fitly exercised in earlier work sees such relations with but little effort. The act of attention to the particular aspect to which attention is solicited is about all that is required.

26. 8¢ is 25% of what my arithmetic cost; my reader cost $87\frac{1}{2}\%$ as much as my arithmetic. Find the cost of the reader.

27. If a 15-ft. line lacks $37\frac{1}{2}\%$ of the length required, what is the length required ?

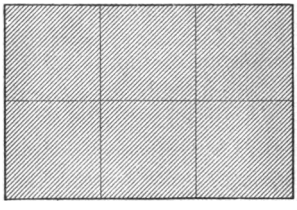
28. The line a equals $12\frac{1}{2}\%$ of the line b. $\frac{1}{2}$ the line a equals what % of $\frac{1}{2}$ the line b? A line 3 times as long as a equals what % of the line 3 times as long as b ?

29. The weight of a 1-in. iron cube equals what % of the weight of a 2-in. iron cube ?

[1] See note, p. 73, "Elementary Arithmetic."

Place before pupils solids pictured on page 77.

1. Call *a* 1, and name the others.
2. $\frac{1}{6}$ equals what % of $\frac{1}{3}$? of $\frac{1}{2}$? of $\frac{2}{3}$? of $\frac{5}{6}$?
3. $\frac{1}{2}$ equals what % of $\frac{5}{6}$? of $\frac{2}{3}$? of $\frac{1}{3}$?
4. $\frac{2}{3}$ equals what % of $\frac{5}{6}$? of $\frac{1}{2}$?
5. 1 equals what % of $\frac{2}{3}$? of $\frac{5}{6}$?



1. Observe the rectangle.

What % of $\frac{1}{3}$ is $\frac{1}{6}$? of $\frac{1}{2}$ is $\frac{1}{3}$? of $\frac{2}{3}$ is $\frac{1}{2}$? of $\frac{5}{6}$ is $\frac{2}{3}$? of $\frac{1}{6}$ is $\frac{1}{3}$? of $\frac{1}{3}$ is $\frac{1}{2}$? of $\frac{1}{2}$ is $\frac{2}{3}$? of $\frac{2}{3}$ is $\frac{5}{6}$? of $\frac{5}{6}$ is 1?

2. $\frac{1}{6}$ equals how many hundredths of $\frac{1}{3}$? of $\frac{5}{6}$?

3. How many hundredths of $\frac{1}{3}$ is $\frac{1}{6}$? of $\frac{1}{2}$ is $\frac{1}{3}$? of $\frac{2}{3}$ is $\frac{1}{2}$? of $\frac{5}{6}$ is $\frac{2}{3}$? of 1 is $\frac{5}{6}$? of $\frac{1}{6}$ is $\frac{1}{3}$? of $\frac{1}{3}$ is $\frac{1}{2}$? of $\frac{1}{2}$ is $\frac{2}{3}$? of $\frac{2}{3}$ is $\frac{5}{6}$? of $\frac{5}{6}$ is 1?

4. $\frac{2}{3}$ is what % of each of the different numbers of sixths in $\frac{6}{6}$?

5. Of what is each of the following $16\frac{2}{3}$ %? 2 da.; 4 da.; 3 da.; $\$\frac{1}{3}$; $\frac{2}{5}$; \$10; \$6; $\frac{1}{7}$; \$.50; 90 bu.; 12 sheep; 120 sheep; $16\frac{2}{3}¢$; $33\frac{1}{3}$ ft.

6. A teacher bought 60 pencils. $16\frac{2}{3}$% of them were lead and the rest were slate pencils. How many slate pencils did she buy?

7. A man pays \$20 for house rent. The rent equals $16\frac{2}{3}$% of his salary. What is his salary?

8. A boy picks 24 qt. of berries, and receives for picking $33\frac{1}{3}$% of the berries. How many quarts does he receive?

9. In a school of 200 pupils $\frac{1}{3}$ of $\frac{1}{2}$ are drawing and the others reading. What % of the pupils are drawing? What reading? The number reading equals what % of the number drawing?

10. I pay \$3 for one book and $66\frac{2}{3}\%$ of \$2 for another. How much do both cost?

11. $\frac{1}{2}$ equals $33\frac{1}{3}\%$ of what? $\frac{1}{2}$ equals $16\frac{2}{3}\%$ less than what part of 100%?

12. A man invests $\frac{5}{6}$ of his money in land. What % does he invest?

13. Show by drawing the ratio of 20% to $\frac{1}{2}$ of 100%.

14. The area of a rectangle $6'' \times 1''$ equals what % of the area of the rectangle $3'' \times 1''$? What is the ratio of $\frac{5}{6}$ of each rectangle to itself? to the other rectangle? of 100% of each rectangle to itself? to the other rectangle?

15. What % of 1 is $\frac{2}{3}$? of $\frac{2}{3}$ is 1?

16. $\frac{1}{2}$ of $66\frac{2}{3}\%$ of the length of a sidewalk equals how many thirds of it?

17. I buy a coat for \$12 and sell it for \$10. What is the ratio of the selling price to the cost? If I sell at a profit of $16\frac{2}{3}\%$, what shall I receive for the coat?

18. The rectangle x equals $16\frac{2}{3}\%$ of the rectangle y. What is the ratio of y to x?

19. The difference between the rectangles equals what % of x? of y?

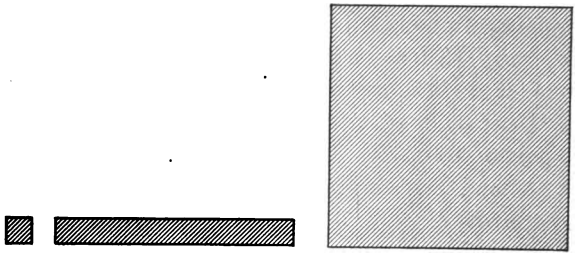
20. A boy made a cent on every 5 papers he sold. What was his % profit if the papers cost $\frac{1}{2}$¢ each? If he sells 60 papers how much does he make?

21. A merchant sells flour for \$5.50 which cost \$4.00. What % is gained on each barrel? on 100 barrels?

22. A field $\frac{1}{2}$ mile square equals what % of a square mile?

23. A man owned a section of land. He sold the S. E. $\frac{1}{4}$ of the N. W. $\frac{1}{4}$. What % did he sell?

Draw a square on the blackboard 1″ long, a rectangle 1″ by 10″ long, and a square 10″ long.



1. What ratios do you see?

2. If c is 100%, what is b? What is a?

3. Show me different parts of c and tell me what % you have shown me. *Ex.* This is $\frac{1}{2}$ of c or 50% of c.

4. 1% of a unit equals what part of it? 1% of the blackboard equals what part of it?

5. What is the ratio of a to $\frac{3}{4}$ of a? of c to $\frac{3}{4}$ of c? of b to $\frac{3}{4}$ of b?

6. If c is 100%, what is $\frac{1}{2}$ of b?

7. If c is 1.00, what is b? a?

Represent by drawing: .03; .07; .05; .02; .08; .37.

8. What % of \$300 is a, if c is \$300? If c is \$300, what % of \$300 is a unit equal to 7 a's?

9. If c is 100%, what is a? What % is $\frac{3}{4}$ of a? Show me $\frac{3}{4}$%.

What is the ratio of 1% to $\frac{3}{4}$%? of 10% to $\frac{3}{4}$%? of 100% to $\frac{3}{4}$%?

10. What is the ratio of 1% to $\frac{1}{8}$%? of 10% to $\frac{1}{8}$%?

What is the ratio of 100% to $\frac{1}{8}$%?

Ans. $100 \cdot \frac{8}{7}$ is the ratio of 100% to $\frac{1}{8}$%.

11. Give as above the ratio of 100% to $\frac{2}{3}$%? to $\frac{3}{8}$%? to $\frac{2}{5}$%? to $\frac{1}{2}$%? to $\frac{1}{5}$%? to $\frac{1}{4}$%?

1. $\frac{3}{4}$ is the ratio of what to 12? What is the ratio of 9 to 12? $\frac{3}{4}$ of 12 equals what? $\frac{3}{4}$ of 9 equals what?

What is the ratio of the sum of 9 and 12 to 12? to 9? $\frac{7}{3}$ is the ratio of what to 9? $\frac{3}{7}$ is the ratio of 9 to what? What is the ratio of the difference between 9 and 12 to each? What is the ratio of each to the difference?

2. If $\frac{3}{5}$ is the ratio of x to y, what is the ratio of y to x? What is the ratio of their sum to y? of their sum to x? What is the ratio of y to the sum? of x to the sum? What is the ratio of their difference to y? to x? What is the ratio of y to their difference?

3. If 75% is the ratio of a to b, what is the ratio of b to a? What is the ratio of their sum to a? to b? What is the ratio of a to the sum? to b? of b to the sum? What is the ratio of their difference to a? to b? of a to the difference? of b to the difference?

4. Divide the blackboard so that 50% of one part equals $33\frac{1}{3}$% of the other; $66\frac{2}{3}$% of the other. What is the ratio of the blackboard to each part?

5. If 50% of the rectangle a equals $33\frac{1}{3}$% of the rectangle b, what is the ratio of a to b?

6. $66\frac{2}{3}$% of the line a equals 75% of the line b. What is the ratio of a to b?

7. $66\frac{2}{3}$% equals the ratio of a to b. What equals the ratio of $\frac{1}{4}$ of a to $\frac{1}{4}$ of b?

What % of a added to a equals b? What % of b should be added to a to make the sum equal to b? The sum of a and b equals how many sixths of b? What % of b? of a?

8. Draw a line and call it 10 yd. Divide it into two parts so that the relative length of the parts shall equal that of 50% of a yd. to 75% of a yd. What is the length of each part?

9. Two boys pay 24¢ for a book. The amount one

pays equals $33\frac{1}{3}\%$ of the amount the other pays. What does each pay?

10. When the time past noon equals $33\frac{1}{3}\%$ of the time to midnight, what is the time?

11.[1] What is the ratio of m to $66\frac{2}{3}\%$ of m? If $66\frac{2}{3}\%$ of m equals k, m equals what $\%$ of k?

12. d equals what $\%$ of $\frac{3}{4}$ of d? If $\frac{3}{4}$ of d equals y, d equals what $\%$ of y?

13. The ratio of a to d is 100; of c to d $\frac{3}{4}$. What is the ratio of a to c?

14. Give problems similar to above.

15. $12\frac{1}{2}\%$ of x equals $16\frac{2}{3}\%$ of b. What is the ratio of x to b?

16. If 75% of a equals 50% of b, a equals what $\%$ of b? Give similar problems.

1. If m is 100%, what is $m-n$? If m is 100%, n equals what $\%$?

2. $\frac{4}{5}$ is the ratio of a to b. Draw a and b. a is what $\%$ more than b? b is what $\%$ less than a?

3. I have 25% more money than you. My money equals what $\%$ of yours? Yours equals what $\%$ of mine?

Represent by rectangles the conditions showing ratios sought in problem 4.

4. Gained 20% by a sale of goods. What was the ratio of the cost to the selling price? of the selling price to the cost?

[1] In none of this work permit pupils to become lost in a maze of words. To say "since" and "therefore" is no evidence of mental action. Turn to the concrete whenever there is hesitation, but see that there is sufficient variety in things used to keep the mind active.

5. If x equals $\frac{1}{2}$ more than y, what part of x equals y? What is the ratio of x to y? of y to x?

Draw figures having these ratios:

6. If the rectangle k equals $\frac{1}{4}$ more than the rectangle h, what part of the rectangle k equals the rectangle h? The difference between the two rectangles equals what part of k? what part of h? what % of each?

7. Charles drew a line and then erased $33\frac{1}{3}\%$ of it. The length of the remaining line was 20 inches. What was the length of the line first drawn?

8. What number diminished by a sum equal to $33\frac{1}{3}\%$ of itself equals 20?

9. 9 minutes is 25% less than the time required to sweep room G. What is the time required to sweep room G?

10. If m equals .3 more than n, what is the ratio of m to n? of n to m?

11. If d is $12\frac{1}{2}\%$ less than a, what is the ratio of a to d? of d to a? of a to the sum of a and d? of the sum to a? to d?

Change the next seven problems, thus:

12. 18 equals $\frac{1}{5}$ more than what unit?
Change to, What is the ratio of the unit to 18?
13. 36 equals $\frac{3}{7}$ more than what unit?
14. 21 equals $\frac{1}{8}$ less than what unit?
15. 192 ft. equals $\frac{3}{5}$ less than what unit?
16. 24 equals .3 more than what unit?
17. \$49 equals .3 less than how many dollars?
18. 2420 equals % more than what unit?
19. 2420 equals % less than what unit?
20. Answer questions from 12 to 19, thus: 18 equals $\frac{1}{5}$ more than 15.

Change the form of the following questions, thus:

1. If 20% be added to Henry's age, the sum will be 24 yr. How old is he?

Change to, If 24 yr. equal 120% of Henry's age, how old is he?

What is the ratio of Henry's age to 24 years? What is his age?

2. A coal dealer in selling coal at \$9 a ton received 20% more than it cost him. What did it cost him? What is the ratio of 100% to 120%?

3. A merchant sold cloth at an advance of $33\frac{1}{3}\%$ on the cost, receiving \$1 per yd. What was the cost per yd.?

4. A grocer sold sugar at 4¢ per lb. and lost 20% of the cost. What did it cost?

5. A mason built 27 ft. of wall, which was 25% less than his contract specified. How much wall did he contract to build?

6. 42 ft. is 27% less than the height of a tree. What is its height?

7. The number of pupils in daily attendance at a school is 370, which is 17% less than the number enrolled. How many pupils are enrolled?

8. A boy spent 40% of his money for a ball and 25% for marbles and had 70¢ left. How much money had he at first?

9. If x, less the sum of 20 % and 70% of x, equals m, what is the ratio of x to m?

10. Sold a horse for \$180, and gained $\frac{1}{8}$ of its cost. What was its cost?

11. A table was sold for \$18, which was $\frac{1}{7}$ less than its value. What was its value?

12. A man sold his house for \$2500 and lost $12\frac{1}{2}\%$. What did the house cost?

13. State the equality of ratios in each problem from 1 to 12.

Ex. The ratio of 100% to $87\frac{1}{2}\%$ equals the ratio of the cost of the house to $2500.

14. m equals the gain when berries are sold at a profit of 20%. What is the ratio of the cost to m? to the selling price?

1. I paid $80 for a buggy and sold it for $16\frac{2}{3}\%$ more than it cost. How much did I receive for it?

2. The bread made from a barrel of flour weighs $37\frac{1}{2}\%$ more than the flour. What is the weight of the bread?

The weight of the bread equals what % of the weight of the flour?

The weight of the bread equals how many eighths of the weight of the flour?

3. Of what are 15, $7\frac{1}{2}$, $9\frac{1}{4}$, 72 and 18, respectively, 25%? Of what are 13, $\frac{1}{2}$, $\frac{3}{7}$, $2\frac{1}{4}$, respectively, $16\frac{2}{3}\%$?

4. A fruit dealer sold pears at a profit of $2 a barrel, which was a gain of 20%. What did they cost?

5. If a miller takes 4 qt. for every bu. of grain he grinds, what % does he take?

6. In a bag of 27 marbles, 6 are white. What % of the marbles are dark?

7. If for the use of $200 for 1 yr. a man pays a sum equal to .05 of $200, what does he pay at this rate for the use of $200 for 4 yr.?

8. At 6% per annum what part of $80 equals the interest of $80 for 2 yr. 8 mo.?

9. $4 is paid for the use of $50 for 1 yr. What is the rate %? What is the ratio of $4 to $50? Express in hundredths.

10. A block $2'' \times 2'' \times 1''$ equals what % of a block $2'' \times 3'' \times 4''$?

1. A lad collected \$200 worth of bills at 6% commission. How much was his commission?

2. A broker sold a lot for \$400 and received a $4\frac{1}{2}\%$ commission. How much was his commission?

3. If I pay an agent 3% commission, what is the cost of every dollar's worth of goods bought? Why?

4. A lawyer received \$50 for collecting a note of \$500. What was his rate of commission?

5. A broker sold a farm for \$6000 and charged \$300 commission. What was the rate of commission?

6. A store worth \$3000 was insured for an amount equal to .01 of its value. How much was the premium?

7. What is the premium for insuring a house worth \$500 at 3%?

8. What must be paid for insuring a building valued at \$3000, for $\frac{2}{3}$ of its value, at $2\frac{1}{2}\%$?

9. The premium for insuring a library worth \$300 was \$6. What was the rate?

10. When the premium is \$6 and the rate 3%, what is the value of the thing insured?

1. A merchant imported 200 yd. of silk, invoiced at \$1.50 per yd. How much was the duty at 20%?

2. A man has real estate worth \$1000 on which he is taxed $1\frac{1}{2}\%$. How much is his tax?

3. If the rate is 5 mills on a dollar, and the tax \$70, what is the amount taxed?

4. If a man's property is assessed at \$14,000, and he pays \$70 tax, what is the rate?

5. A man bought butter at 15¢ a lb. and sold it for 25¢ a lb. What % did he gain? If he bought it at 25¢ a lb. and sold it at 15¢ a lb., what % did he lose?

6. $12\frac{1}{2}$ % of 96 equals $83\frac{1}{3}\%$ of what number?

7. What number equals $\frac{8}{7}$ of 20% of 80?

8. $66\frac{2}{3}\%$ of \$120 equals $\frac{4}{5}$ of what was paid for a watch. How much did the watch cost?

9. A watch bought for \$120 was sold for \$90. What % was lost?

10. Make statements similar to the following: By selling gingham for 60¢, I gain 25%. The ratio of the selling price to the cost is $\frac{4}{5}$. The cost is 48¢.

Wall paper selling for 16¢ per roll brings $33\frac{1}{3}\%$ profit. The ratio of the selling price to the cost is $\frac{4}{3}$; of the cost to the selling price is $\frac{3}{4}$. The cost is 12¢.

11. Sold pine slabs for \$3 and lost $16\frac{2}{3}\%$. The selling price equaled how many sixths of the cost? What was the cost?

12. Make statements similar to the following: Sold paints for \$1125 and gained 25%. $\frac{5}{4}$ is the ratio of \$1125 to the cost. The cost was $\frac{4}{5}$ of \$1125, or \$900.

13. Make problems about % of gain or loss in buying and selling coffee, sugar, butter, tea, apples, oranges, milk, ice, chinaware, table linen.

14. Pineapples costing 30¢ were sold at 20% profit. For how much were they sold?

15. A book bought at \$1.25 was sold at a profit of 30%. For how much was it sold?

16. If 20% is the gain on peaches sold at \$2.40, what is the cost? If selling at \$2.40 is selling at a loss of 25%, what is the cost?

17. There is a loss of $37\frac{1}{2}\%$ on damaged silk sold at \$1.25 a yd. What is the cost? What is the loss on 40 yd.? What is the % of loss on 40 yd.?

18. When the ratio of loss to the cost of goods is $12\frac{1}{2}\%$, the selling price equals what per cent of the cost? What is the ratio of the cost to the selling price?

19. A boy sold a bicycle for \$30, which equaled 80% of the cost. What was the cost?

1. A man deposited \$200 in a bank and each year increased his deposit 100%. How much was his deposit at the end of 3 years?

2. Tea that cost \$1.20 was sold at a profit of $12\frac{1}{2}\%$. What was the selling price? What is the ratio of the selling price to \$1.20?

3. At how much per yd. must cloth that costs 20¢ a yd. be sold to gain 20%? cloth that costs 25¢ a yd.? that costs 35¢ a yd.?

4. What will eight \$100 shares of telegraph stock cost at 4% premium? at 4% discount?

5. A man bought 10 shares of nursery stock at par and sold them at a premium of 6%. What was his profit?

6. A dividend of 8% is declared. How much does a stockholder who owns three \$100 shares receive?

7. The ratio of the rectangle a to the rectangle b is $\frac{5}{3}$. The difference between the rectangles equals what % of b? of a?

If the sum of the areas of the two rectangles is 22 sq. in., what is the area of each rectangle?

8. A merchant sold goods for a sum equal to $\frac{11}{10}$ of the cost and gained \$4.40. What was the cost?

9. A grocer sold nuts at a profit of $16\frac{2}{3}\%$ and gained 42¢. What was their cost?

10. A stationer buys paper at \$1 a ream and sells it at $\frac{1}{2}$¢ a sheet. Does he gain or lose? What %?

11. If $\frac{1}{2}$ acre is sold for what an acre cost, what % is gained?

12. If $\frac{1}{2}$ of a quantity is sold for a sum equal to the cost of $\frac{3}{4}$ of it, what % is gained?

13. If $\frac{4}{5}$ of a quantity is sold for a sum equal to $\frac{1}{2}$ the cost of the whole, what % is lost?

14. $\frac{2}{3}$ of the sum received for an article equals $\frac{5}{6}$ of what was paid for it. What is the gain %? What is the ratio of the sum received to the cost? Show by drawing.

15. By a mistake in weighing, an amount of coal so much less than a ton was sold for a ton that the seller gained 8%. What part of a ton was sold for a ton? What % did the buyer lose? Show conditions by drawing.

16. Goods that cost $x¢$ a yd. are marked to sell at 25% profit, but are sold at 25% less than marked price. Draw rectangles showing ratio of marked price to cost price; of selling price to marked price; to cost price.

17. Make many statements similar to the following: Goods marked to sell at $33\frac{1}{3}\%$ profit were sold at 50% below marked price. The ratio of the selling price to the cost is $\frac{2}{3}$.

18. To sell at cost, goods marked at 25% profit should be sold for what % below marked price? goods marked at $33\frac{1}{3}\%$ profit? 20%? $12\frac{1}{2}\%$?

19. If goods are marked at 50% profit, at how much below marked price must they be sold to gain 10%? 10% below marked price at 25% profit equals what part of the cost? What is the ratio of 40% to 150%?

20. A dealer lost 8% of a box of fruit. At what % above cost must he sell that he may lose nothing? that he may gain 12%?

By no manipulation of figures and formulas can the pupil arrive at truth concerning things. Sensible experience is in all cases the basis. Gauss called geometry "a science of the eye," as Thiersch had before called work in elementary mathematics a "thinking with the eye."

1. What is the ratio of 110% to 100%? How much currency can be obtained for $50 in gold when gold is at a premium of 10%? 100%? 150%?

2. 1.06 is the ratio of what to 100%? When stock is at 6% premium, what is the market value of $1? of $100? of $700?

3. When stock is at $16\frac{2}{3}\%$ discount, what is the market value of stock whose par value is \$40? \$100? \$1000? What ratios are equal?

4. Sold muslin at 45¢ a yd. and gained $12\frac{1}{2}\%$. How much did it cost?

5. Find the cost of coal sold at \$7, the loss being $12\frac{1}{2}\%$.

6. 25% of 800 bu. equals $12\frac{1}{2}\%$ of how many bushels?

7. A man sold goods at a gain of 15%. His profit was \$60. For how much did he sell them?

8. What is the amount of sales when the commission at $1\frac{1}{2}\%$ is \$300?

9. A man sold a house at a profit of \$360, which was 6% more than it cost him. How much did he pay for the house? For how much did he sell it?

10. What will 10 shares of stock cost at 10% below par, if I pay a broker $\frac{1}{2}\%$ for buying, or brokerage?

11. What is the annual income on a bond of \$4000 which yields 6% annually?

1. What is the interest on \$70 for 1 yr. at 8%? for 2 yr.? for $1\frac{1}{2}$ yr.? for 6 mo.?

2. When \$300 is loaned at 7% per yr., what is the amount of the principal and interest in $1\frac{1}{2}$ yr.?

3. When the interest on \$100 for 1 yr. is \$7, what is the rate?

4. When the interest on \$100 for 2 yr. is \$16, what is the rate?

5. What principal at 6% yields \$300 in 1 yr.?

6. What principal yields \$350 in $3\frac{1}{2}$ yr. at 10%? What $\%$ does any principal earn in $3\frac{1}{2}$ yr. at 10%?

7. At what $\%$ does \$75 in 2 yr. amount to \$91? How much interest does the \$75 yield in 2 yr.?

8. At what $\%$ per annum does \$50 amount to \$65 in $2\frac{1}{2}$ yr.?

9. At what rate does \$500 gain \$50 in 1 yr.? in 2 yr.? in $1\frac{1}{2}$ yr.?

10. If \$800 yields \$120 in $2\frac{1}{2}$ yr., what does it yield in 1 yr.? What is the rate?

11. At what % does \$900 in 3 yr. yield \$180 interest?

12. At what rate per annum does \$100 gain \$25 in $4\frac{1}{2}$ yr.? What part of the \$25 is gained in 1 yr.?

13. At what % per annum does a principal double itself in 4 yr.? in 5 yr.? in 9 yr.?

14. At what rate must \$320 be loaned to yield \$16 in 1 yr.?

1. If 5% is the ratio of h to k, what is the ratio of k to h? of their sum to k? of their sum to h? of their difference to k? of their difference to h? of k to their difference? of h to their difference?

2. .08 is the ratio of what unit to 75 lb.? What is the ratio of 6 lb. to 75 lb.? If .08 is the ratio of 6 lb. to 75 lb., what is the ratio of 75 lb. to 6 lb.? of their sum to 75 lb.? of their sum to 6 lb.? of their difference to 75 lb.? of their difference to 6 lb.?

3. What equals 35% of 640 acres?

$$\frac{35 \cdot 640}{100} = 224.$$

$\therefore$ 224 acres equals 35% of 640 acres.

4. 224 acres equals 35% of what?

$$\frac{100 \cdot 224}{35} = 640.$$

$\therefore$ 224 acres equals 35% of 640 acres.

5. 224 acres equals what % of 640 acres?

$$\frac{224 \cdot 100}{640} = 35.$$

640 acres equal what % of 640 acres? What is the ratio of 224 acres to 640 acres? To what part, then, of 100% of 640 acres is 224 acres equal?

∴ 224 acres equal 35% of 640 acres.

6. 35% is the ratio of what to 640 acres? What is the ratio of the sum of 640 acres and 224 acres to 640 acres? to 224 acres? What is the ratio of 640 acres to the sum? of 224 acres to the sum?

1. What is 6% of 75 bu.?

$$\frac{6 \cdot 75}{100} = ?$$

To Teacher. — Ask many questions similar to the following: 6% is the ratio of what to 75 bu.? What is the ratio of 75 bu. to 4.5 bu.?

2. What is 25% of 160 ft.?
3. What is $37\frac{1}{2}$% of \$845?
4. What is 8% of 647 oz.?
5. What is $\frac{3}{4}$% of \$824? (What is $\frac{3}{4}$ of 1% of \$824?)

$$\frac{3 \cdot 824}{4 \cdot 100} = ?$$

What equals 1% of \$824? What, then, equals $\frac{3}{4}$ of 1% of 824?

6. What is $\frac{5}{8}$% of 876 ft.?
7. What equals $\frac{6}{7}$% of \$214?
8. What equals $\frac{7}{4}$% of 64.82 tons?
9. What equals $2\frac{1}{2}$% of \$78?

$$\frac{5 \cdot 78}{2 \cdot 100} = ?$$

What is the ratio of $2\frac{1}{2}$% of \$78 to 1% of \$78? What equals 1% of \$78? What, then, equals $\frac{5}{2}$% of \$78?

10. What is $3\frac{1}{3}\%$ of 729 days?
11. What equals $7\frac{1}{4}\%$ of \$856.50?
12. What equals $3\frac{5}{8}\%$ of 1600 ft.?
13. What equals $13\frac{1}{2}\%$ of \$1672.34?
14. What equals $87\frac{1}{2}\%$ of 647 yd.?
15. What equals $66\frac{2}{3}\%$ of 5876?
16. What equals $37\frac{1}{2}\%$ of 643?
17. Make 10 sentences like this: $\frac{2}{7}$ of $\frac{1}{100}$ of 745 da. equals $3\frac{3}{7}\%$ of 745 da.
18. Make 10 sentences like this: $\frac{5}{8}$ of 940 ft. equals $62\frac{1}{2}\%$ of 940 ft.

1. What is the ratio of 6 mo. to 1 yr.? of 9 mo.? of 4 mo.? of 8 mo.? of 1 yr. 4 mo.? of 1 yr. 6 mo.? of 1 yr. 7 mo.? of $2\frac{1}{2}$ mo.? of 1 yr. $2\frac{1}{2}$ mo.?
2. If 7% of \$$y$ equal the interest of \$$y$ for 1 yr., what equals the interest of \$$y$ for 6 mo.? for 1 yr. 6 mo.? for 1 yr. 8 mo.? for 1 yr. 5 mo.?
3. If \$84 is the interest of \$$y$ for 1 yr. 9 mo., what is the interest of \$$y$ for 1 yr.?
4. If \$$x$ at 8% yields \$92 in 1 yr., what does it yield at the same rate in 9 mo.? in 1 yr. 8 mo.?
5. What part of \$640 equals the interest for 1 yr. at 7%? What part, then, of $\frac{7}{100}$ of \$640 equals the interest for 1 yr. 8 mo. at 7%?
6. What is the interest of \$640 for 1 yr. 8 mo. at 7%?

$\frac{5 \cdot 7 \cdot 640}{3 \cdot 100} = ?$

$\frac{7}{100}$ of \$640 equals the interest for what time?

$\frac{5}{3}$ of $\frac{7}{100}$ of \$640 equals the interest for what time?

7. What is the interest of \$600 for 1 yr. 3 mo. at 9%?
8. \$850 for 2 yr. 7 mo. at 10%?
9. \$1270 for 3 yr. 4 mo. at 8%?
10. \$500 for 7 mo. at 7%?

11. $1250 for 3 yr. 6 mo. at 7%?
12. $200 for 9 mo. at 7½%? Of $200 for 1 yr. at 4¾%?

1. ⅝ is the ratio of what to $25? What is the ratio of $30 to $25? of $25 to $30? of their sum to $30? of their sum to $25?

2. 1⅓ is the ratio of what to 15¢? What is the ratio of $5 to 15¢? of 15¢ to $5? of their sum to 15¢?

3. If 1⅐ is the ratio of x to y, what is the ratio of y to x? of their sum to y? of their sum to x? of y to their sum? of x to their sum?

4. 168 men equals 8% of how many men?

$\frac{100 \cdot 168}{8} = ?$ The ratio of 100% to 8% equals the ratio of how many men to 168 men?

5. Of what unit is 27.5 bu. 7%?
6. Of what unit is 73 cd. 16%?
7. 255 equals 30% of what unit?
8. 180 equals 12½% of what unit?
9. $220.50 equals 107% of what unit?
10. $75 equals 104% of what unit?
11. 231 gal. equals 2½% of what?

$\frac{100 \cdot 2 \cdot 231}{5} = ?$ What part of 231 gal. equals 1%? What, then, equals 100%?

12. $846 equals 1½% of what unit?
13. 854.37 ft. equals 5⅓% of what unit?
14. 247 yd. equals 2⅖% of what unit?
15. $675.25 equals 2⅜% of what unit?
16. $785.56 equals 7⅝% of what unit?
17. 68 ft. equals ⅜% of what unit?
18. 2745 equals ⅝% of what?
19. Of what unit is 45 ⅜%?
20. Of what unit is 144 37½%?
21. 210 equals 87½% of what?
22. 80 equals 66⅔% of what unit?

23. Of what unit is \$745 $83\frac{1}{3}\%$?

24. Write ten sentences similar to this: If 594 equals $3\frac{1}{2}\%$ of a unit, $100 \cdot \frac{1}{7}$ of 594 equals the unit.

25. Write ten sentences similar to this: If 69 ft. equals $66\frac{2}{3}\%$ of a unit, $\frac{3}{2}$ of 69 ft. equals the unit.

1. Express the ratio of 4 to 6 in different terms; of 15 to 25; of 7 to 10; of 2 to 50.

$\frac{a}{b}$

2. What is the ratio of a to b expressed in hundredths?

$\frac{3 \cdot 1.00}{4} = .75.$

b equals how many hundredths of b?
.25
What is the ratio of a to b?
What, then, is the ratio of a to 100 hundredths of b?

3. b equals how many hundredths of a?

$\frac{4 \cdot 1.00}{3} = 1.33\frac{1}{3}.$

a equals how many hundredths of a?
$.33\frac{1}{3}$
What is the ratio of b to a?
What, then, is the ratio of b to 100 hundredths of a?

4. What is the ratio of 9 to 12 expressed in hundredths?

$\frac{9 \cdot 1.00}{12} = ?$

5. What is the ratio of 7 to $2\frac{1}{2}$ expressed in hundredths?

$\frac{14 \cdot 1.00}{5} = ?$ What is the ratio of 7 to $2\frac{1}{2}$?

1. What is the ratio of $2\frac{1}{2}$ to 7?

$\frac{5 \cdot 100}{14} = \frac{250}{7} = 35\frac{5}{7}.$

50
7

If 7 equals 100%, what % does $2\frac{1}{2}$ equal?
If $2\frac{1}{2}$ equals 100%, what does 7 equal?
$2\frac{1}{2}$ equals what % of 7?

$\therefore 2\frac{1}{2}$ equals $35\frac{5}{7}\%$ of 7.

State the % relation of a to b and of b to a:

	a	b	a	b
2.	150 yd.	450 yd.	42.7	.24
3.	560 min.	140 min.	1287	543
4.	720 bu.	370 bu.	$\frac{2}{3}$	$\frac{3}{4}$
5.	75 sheep	300 sheep	$\frac{5}{8}$	$\frac{2}{3}$
6.	350	329	\$5	$\$5\frac{1}{2}$
7.	$8\frac{1}{2}$	12	$\frac{1}{2}$	$\frac{4}{5}$
8.	$85\frac{1}{2}$	72	$\frac{1}{2}$	$\frac{3}{4}$
9.	6.4	1.6	$\frac{4}{5}$	$\frac{1}{2}$

1. $4\frac{1}{3}$ equals what % of $9\frac{3}{4}$?

$\frac{13 \cdot 4 \cdot 100}{3 \cdot 39} = ?$ $9\frac{3}{4}$ equals what % of $9\frac{3}{4}$?

Then what equals the % that 1 is of $9\frac{3}{4}$?

Then what equals the % that $4\frac{1}{3}$ is of $9\frac{3}{4}$?

2. $5\frac{1}{2}$ equals what % of $4\frac{2}{7}$?
3. $4\frac{2}{7}$ equals what % of $5\frac{1}{2}$?
4. $19\frac{1}{2}$ equals what % of $25\frac{3}{5}$?
5. 6.7 equals what % of 54?

1. A man invested \$5280 in cotton and sold it at a profit of 12%. What was his profit?

2. A grocer bought 500 bananas. $37\frac{1}{2}$% of them were not marketable. How many did he sell?

3. A man collected 40% of a note of \$675 and charged $6\frac{1}{4}$% commission. What was his commission? On what did he receive his commission?

4. A man owed \$347 on account and settled it for cash at 2% off. What was the discount?

5. A man paid a tax of \$73.50, which was 3% of the value of his property. What was its value?

6. What was the amount of the sales when the commission paid was \$185.60 and the rate $3\frac{1}{3}$%?

7. At $7\frac{1}{2}\%$ an attorney was paid \$144.06 for collecting a note. What was the face of the note?

8. An agent sells 550 bbl. of flour at \$10 a bbl. and remits \$5000. What is the rate of commission?

9. An army of 1400 men went into a battle; after the battle there were only 900. What was the loss %?

10. A man's expenses are \$500 a year; his income is \$1600 a year. What % of his income equals his expenses?

11. A chain is 14 carats fine. What % of it is gold?

1. If 15% of a certain ore is silver, how much silver is there in 4850 lb. of ore?

2. After a deduction of 8% from a bill of \$416.28 is made, how much is the bill?

3. A boy bought bananas at \$1.50 a hundred and sold them at \$.03 each. What % did he gain?

4. A man bought secondhand books at \$10 a dozen and sold them at \$1.50 each. What was his % of gain?

5. If the income of \$2000 is \$250, what is the rate?

6. After taking out 30% of the grain in the bin there remained 40 bu. $3\frac{1}{3}$ pk. How much wheat was there at first?

7. A broker bought stock at 3% discount, and, selling the same at 5% premium, gained \$560. How many shares, each worth \$100, did he purchase?

8. What sum must I invest in stock, at par, paying an annual dividend of $5\frac{1}{3}$ %, to realize an income of \$2200 yearly?

9. At what rate must I invest a trust fund of \$30,000 to secure an annual income of \$1000? a semi-annual income of \$500?

10. A principal of a school receives \$198 per month, after a deduction of \$2 for pension fund has been made. What % of salary is deducted for pension fund?

1. A man sold 25% of 1000 bu. of potatoes. The remainder was $16\frac{2}{3}\%$ of his entire crop. How much was his crop?

2. \$18 was spent in repairing a carriage which cost \$105. It was then sold for \$160. What was the % of profit?

3. 150 bu. of apples were sold at \$1.25 a bu. What was the rate of gain if the cost was 80¢ a bu.?

4. A merchant marked silk at \$2.75 a yd. and sold it at 10% below the marked price. For how much did he sell it?

5. Bought 150 bbl. of flour for \$850 and sold it at a loss of 16%. What was the selling price per bbl.?

6. If \$60 is the amount paid for insuring 15 horses at $4\frac{1}{2}\%$, what is the value of the horses?

7. A man who owned 70% of a store sold 40% of his share for \$7000. What was the value of his share? What was the value of the store?

8. Cotton was bought at $15\frac{2}{3}$¢ a lb. and sold at $18\frac{1}{2}$¢. What was the gain %?

9. Carpeting cost $\$1.87\frac{1}{2}$ a yd. and was sold for \$2.25 a yd. What was the gain %? What amount must be added to $\$1.87\frac{1}{2}$ that the sum shall be \$2.25?

10. A real estate agent sold 430 acres at \$75 an acre and charged a commission of $3\frac{1}{2}\%$. How much was his commission, and how much was paid to the land owner?

1. Find the cost of 120 shares of N. Y. Central R. R. stock at $87\frac{1}{2}$, brokerage $\frac{1}{2}\%$. What does \$1 of the stock cost? What is the cost of 1 share?

2. A broker bought 40 shares of R. R. stock at 92 and sold them at 105. How much did he gain?

3. A broker received \$4.80 for a draft of \$1280. The brokerage equals what % of the draft?

4. A broker was paid $300 for buying $15,600 worth of stock. The commission equaled what % of the value of the stock?

5. Bought bonds at 112 and sold them at 115, making $300. How many bonds of $1000 each did I buy?

6. If shoes marked at $3.50 a pair are sold at 10% discount, what is the discount? What is the net price?

7. A man asked $125 for a buggy, but for cash took 10% off. How much was deducted?

8. A broker received $112.50 for selling bonds, charging $\frac{1}{8}$% brokerage. For how much were the bonds sold?

9. A man sells goods at a profit of 16% and makes $48. What is the cost?

10. A man bought resin at $2.25 a bbl. and sold it at $5.30 a bbl. What was the gain %?

1. Paid $60 for insuring a house worth $1200 for 2 yr. What was the annual rate?

2. In a school of 59 pupils the average daily attendance was 47. What was the % of attendance?

3. A mill worth $16,000 is insured for $\frac{3}{4}$ of its value at $1\frac{1}{2}$%. What is the premium?

4. How much must be paid for insuring a consignment of goods worth $7840 at $2\frac{1}{4}$%?

5. I insured my house for $\frac{3}{4}$ of its value at $1\frac{1}{3}$% and paid $130. What was the value of the house?

6. A house which was insured for $\frac{3}{4}$ of its value was burned. Its value was $4900. What % did the owner lose?

7. When a premium of $750 is paid, the rate being $1\frac{1}{2}$%, what is the amount insured?

8. Paid $187 to insure $\frac{1}{2}$ the value of a store at $2\frac{3}{4}$%. What was the value of the store?

9. In a school of 50 pupils there were 150 absences in 8 weeks. What was the % of attendance?

10. A cubic foot of water weighs 62½ lb. and a cubic foot of ice 57½ lb. Ice is what % lighter than an equal bulk of water?

1. An agent furnished a schoolhouse for $2000 and received $40 commission. What was the rate %?

2. The rate of tax in a town is 1¾%. How much is Mr. Smith's tax if his property is valued at $10,000?

3. The taxable property of a town is $1,505,470. The rate of tax for school purposes is 3½ mills on the dollar. What is the amount of school tax assessed?

4. If the taxable property of a town is $1,505,470 and the tax collected $5269.145, what is the rate of taxation?

5. What is the cash value of a bill of goods amounting to $497 at 25% discount and 5% off for cash?

6. What is the cash value of a bill of goods amounting to $7845 at 16% discount and 3% off for cash?

7. What is the duty at 20% *ad valorem* on 140 yd. of goods valued at $2 a yd.?

8. What is the duty at 40% *ad valorem* on 600 yd. of silk valued at $1.75 per yd.? For how much a yd. must the importer sell the silk to clear 35%?

9. An attorney received $125 for collecting rents to the amount of $2150. What was the rate of commission?

10. If it costs $93.50 to insure a store for ⅔ of its value at 1⅜%, what is the value of the store?

1. A man borrows money at 6% per annum and pays $48 interest. What more do you need to know in order to find the amount borrowed? If $48 is the interest paid in 6 mo., what was the sum borrowed? if $48 is the interest for 1 yr.? for 2 yr.?

2. If for the use of money for 1 yr. a man pays a sum equal to .07 of what he borrows, in 2 yr. he pays a sum equal to what? in 3 yr.? in 4 yr. and 4 mo.?

3. A sum equal to .05 of $\$m$ is a man's interest for 1 yr. What equals his interest for 2 yr. and 6 mo.?

4. What sum of money lent at 6% yields \$72 interest in 1 yr.?

$\frac{100 \cdot \$72}{6} = \$1200.$ What is the ratio of the principal to the interest for 1 yr.?

5. \$120 is the interest for 1 yr. at 10%. What equals the principal?

6. What sum of money produces \$34 interest in 2 yr. and 6 mo. at 6%?

7. What sum of money produces \$42.35 interest in 1 yr. and 6 mo. at 7%?

8. What principal earns \$24 in 2 yr. at 6%?

9. How much money has a man loaned if he receives for its use \$12 at the end of 3 yr., interest 5%?

10. At 5% what principal yields \$350 in 6 mo.?

11. What sum of money produces \$25 in 7 mo. at 9%?

12. How much must I invest at 6% for 1 yr. and 9 mo. to earn \$900?

To Teacher. — Require pupils to make problems similar to those given. Select problems and have pupils use them as a basis for new problems. *Ex.* A broker sold property valued at \$112.50, at a commission of 3%. Find his commission.

Make a new problem, thus: What is the interest on \$112.50 at 3%? Or, How much must be paid for insuring clothing worth \$112.50 at 3%? Unless pupils can construct problems similar to those given, and readily make new questions by changing the conditions, their work is mechanical and not mental. There should be growing power to discriminate, to see likeness amidst diversity, to separate the essential from the accidental.

1. An auctioneer sold \$374 worth of furniture and charged a commission of 5%. How much did he receive?

2. At 5% commission an auctioneer received \$18.70 for selling furniture. For how much was the furniture sold?

3. \$18.70 was the commission paid for selling \$374 worth of furniture. What was the rate of commission?

4. Mr. Smith bought a house for \$4850 and paid 15%. How much did he still owe?

5. A merchant sells \$250 worth of goods and gains \$50. What is his rate of profit?

6. What is the interest on \$275 for 1 yr. at 6%?

1. $\frac{1}{4}$ is the ratio of x to y. The sum of x and y is 30. What is the value of each?

2. A boy received 30¢ with which to buy marbles after taking out for making the purchase $\frac{1}{4}$ as much as he invested. How much did he expend for marbles? Show by drawing.

The sum paid for the marbles equals what part of 30¢? Why?

3. The line $\overset{m}{\text{———}}$ represents the amount an agent receives with which to make an investment after deducting his commission of 20%; show the part of the line representing the investment.

4. The amount received equals the sum of the amount invested and what?

"Reasoning and classification are the necessary complements of each other. . . . It follows that, contemplated from this point of view, reasoning is a classification of relations. But what does classification mean? It means the grouping together of those which are like — the separation of the like from the unlike. . . . The idea underlying all classification is that of similarity." — HERBERT SPENCER.

5. The amount an agent receives equals $\frac{8}{7}$ of the amount invested. The amount invested equals what part of the amount received?

6. An agent receives $\frac{1}{10}\frac{0}{0}\frac{3}{3}$ of the amount he invests. What is the ratio of the amount invested to the amount received?

7. A man received \$84 with which to buy cotton after deducting for making the purchase $\frac{1}{20}$ as much as he invested. What did he pay for the cotton?

The sum paid for the cotton equals what part of the money received?

8. An agent received \$500 with which to buy wheat, after deducting for making the purchase .05 as much as he invested. What did he pay for the wheat? What is the ratio of the amount the agent received to the amount invested in wheat?

If the agent received .05 of the \$500, on what would he receive commission besides the money invested in wheat?

9. An agent received \$250 with which to buy oranges after deducting his commission at 7%. How much did he expend for oranges?

10. A Chicago merchant sent his agent in New York \$5275.20 to be invested in coffee after deducting his commission at $1\frac{1}{2}$%. How much did he expend for coffee and what was his commission? What is the ratio of the money invested in coffee to the amount the agent received?

11. An agent bought goods at $\frac{3}{4}$% commission; paid \$75 for expenses, and sent a bill for \$2847.50. What was the amount of the purchase?

12. A stock of prints was sold at a commission of $1\frac{1}{2}$%, and the proceeds invested in cambrics was \$35. For how much did the prints sell?

\$35 equals what part of the selling price of the prints?

13. An agent received $\frac{1}{10}$ of the selling price of a horse. The owner received \$90. For how much was the horse sold?

14. For how much must a lot be sold that the owner may receive \$600 after paying a real estate dealer 2% for selling it? The selling price equals how many ninety-eighths of \$600?

15. For how much must a farm be sold that the owner may receive \$5000 and an agent 5% for selling it? What % of the selling price does the owner receive?

What is the ratio of the selling price to \$5000?

16. A man wishes an agent to sell his house, which cost \$5250, for enough to cover both the cost of the house and the agent's commission at 2%. For how much must the house be sold?

\$5250 equals how many hundredths of the selling price?

17. For how much must lumber worth \$27,845 be insured at 3% to cover both the value of the lumber and the premium?

18. For how much must you give a note at a bank to obtain \$180 if the banker retains $\frac{1}{10}$ of the value of the note for the use of the money?

19. If \$250 is the net proceeds of a note of which 2% was retained for the use of the money, what is the face of the note?

20. The amount paid the teachers in a district is \$3740; what amount of tax must be assessed for the teachers' fund if the cost of collecting the tax is 2%?

21. The net proceeds of a tax assessment, after deducting $2\frac{1}{2}$% for collection, was \$8794.75, and 5% of the tax was not collected. What was the assessment?

22. If you buy potatoes at 45¢ a bu. and $\frac{1}{4}$ of them spoil, at what price must you sell the remainder that you may lose nothing?

What part of a bu. must be sold for 45¢ that there may be no loss?

For how much, then, must a bu. be sold?

23. A dealer bought wool at 28¢ per lb. If in cleansing it loses $\frac{1}{3}$, at how much per lb. must he sell the clean wool to gain $\frac{1}{7}$ on the cost?

24. A fruit dealer lost 25% of a quantity of apples and sold the remainder at a gain of 33$\frac{1}{3}$%. Required the % of gain or loss.

25. How many yd. of ducking, $\frac{3}{4}$ of a yd. wide, are required to line the carpet of a room 12 ft. by 15 ft., if the ducking shrinks 4%?

26. Mr. Brown paid $\$x$ for a horse and sold it for 20% more than he paid and $\frac{1}{4}$ less than he asked for it.

Represent cost, selling price, and asking price.

27. A man bought a horse for \$72 and sold it for $\frac{1}{4}$ more than it cost and $\frac{1}{10}$ less than he asked for it. How much did he ask for it?

28. Find the marking price of goods that cost \$15 so as to reduce the marked price $\frac{1}{4}$ and yet make a profit of $\frac{1}{3}$.

29. Find the marking price of goods that cost \$60 so as to sell 10% below the marked price and still gain 20%.

30. A man sold a buggy for 20% less than he asked for it and received \$95, which was 20% more than it cost. What was the cost and what was the selling price?

31. An agent receives a discount of 50% from the retail price of articles and sells them at the retail price. What is his gain?

32. What must I ask for a horse in order to fall 33$\frac{1}{3}$% and still make 33$\frac{1}{3}$%?

33. A merchant asked 25% more for goods than they cost him, but at last sold them at a reduction of 25% from his asking price. If the cost of the goods was \$150, did he gain or lose, and what %?

34. $3745 less the agent's commission at 3% equals the sum Mr. A invested in wheat. How much was the agent's commission?

35. At 2% commission an agent received $282 for the purchase of apples at $3 a bbl. How many bbl. of apples did he purchase?

36. The cost of a horse and saddle is $180; 20% of the cost of the horse equals the cost of the saddle. What is the cost of each?

37. 20% of a pole is in the mud, 33⅓% in the water, and 14 ft. in the air. How long is the part in the mud and water?

38. A man gave $150 for a watch and chain. The cost of the chain was equal to 25% of the cost of the watch. How much did each cost?

39. A and B do a piece of work in 10 days. A does 66⅔% as much work as B. What part of the work does each do in 10 days? How long would it take each to do the work alone?

40. Henry's money equals 33⅓% more than Harry's. What is the ratio of Harry's to Henry's? How many % is Harry's less than Henry's?

41. If the selling price of goods is ⅓ less than the asking price, the ⅓ is equal to what part of the selling price?

42. If the 4-ft. wood which a dealer sells proves to be 4 in. short, what % of a cd. does a buyer lose? What % does the dealer make?

43. A builder who charged 3% for superintending the construction of a bridge received $246 commission. What was the actual cost of the bridge?

44. Of a mixture of milk and water ⅓ is water. How much of the mixture would you need to buy to obtain a gal. of milk?

If 4% of the mixture were water, how much must be purchased to obtain a gal. of milk.

45. A commission merchant received 3% for selling and 2% for buying. After selling a consignment of goods and taking out both commissions he invests the proceeds. If the entire commission is $87, what amount did he invest? What is the ratio of the 3% commission to the amount after deducting it?

1. When land is selling at an advance of $20 an acre, what is the gain % if it costs $70 an acre?

2. A man paid an agent 5% for selling a house and received $5824. For how much did the agent sell the house?

3. A man collects debts to the amount of $375.80. How much is his commission at 4½%?

4. Mr. Wilson rents 160 acres of land at $2 an acre. If the land is worth $25 an acre, the rent equals what % of the value of the land?

5. On what sum do taxes at ⅓% amount to $200?

6. If an investment of $624 yields me $43.68 per annum, what is my rate of profit?

7. A man bought sugar at $9 per cwt. and sold it at a loss of 4%. For how much did he sell it?

8. A grocer bought 8 doz. eggs at 10¢ a doz. and 8 doz. at 5¢ a doz. He sold them all at 15¢ a doz. What was his % of gain?

9. If oranges are bought at 20¢ and sold at 30¢ a doz., how many must be sold for a man to realize 50% on an investment of $10?

10. If by selling wine at $2.70 a qt. I lose 10%, at what price must I sell it to gain 10%?

11. I bought $5000 worth of stock at 95½. At what price must I sell it to gain 10%?

12. A and B earned $600. A is to receive 10% less than B. Find the share of each.

13. If I lose 10% by selling goods at 40¢ per yd., at what price should I sell to gain 20% ?

14. If a man buys a house for $6840 and receives $1250 for rent in 2 yr. and 3 mo., what rate of interest does the investment yield ?

15. If I sell $\frac{2}{3}$ of an acre of land for an amount equal to what I paid for 1 acre, what % do I make ?

16. If I sell $\frac{3}{4}$ of a quantity for an amount equal to what $\frac{3}{8}$ of it cost, what is my loss % ?

17. In what time does $600 at 7% per annum produce $105.20 interest ?

18. If I gain 40% by selling wood at $6.40 a cd., how much did it cost me ?

19. Paid $8000 for stocks at 16% below par and sold at 112. What % was gained ?

20. A man performs a piece of work in 8 days, and a boy does an equal amount in 12 days. The amount of work the boy does in 1 day is what % of the amount the man does in the same time ?

21. A man sold two lots for $500 each, gaining 20% on one and losing 20% on the other. Did he gain or lose, and how much ?

22. Bought a horse and carriage for $390; the cost of the carriage was equal to 30% of the cost of the horse. Find the cost of each.

23. For what sum must goods worth $6750 be insured at $3\frac{1}{2}$% to cover both property and premium ?

24. What was received for a sale of goods marked at $42.40, at 6% discount, and 5% off for cash ?

25. A man's taxes are $75.60, and this equals $2\frac{1}{2}$% of the value of his property. What is its value ?

26. The number of peach trees in an orchard is 20% more than the number of apple trees. The number of apple trees is how many % less than the number of peach trees ?

27. An agent receives $210 to invest in corn after deducting his commission at 5%. How much corn does he buy at 50¢ a bu.?

28. An agent received a sum of money to invest in flour, after taking out his commission at 8%. If he invested $160 in flour, how much did he receive?

29. An agent sold two lots for $1000 each. On one he gained 5%, on the other he lost 5%. Did he gain or lose, and how much?

Interest. — 1. What part of any principal equals the interest for 1 yr. at 25%? at $33\frac{1}{3}$%? at 7%? at 10%? at $8\frac{1}{2}$%?

What is the ratio in each case of the principal to the interest?

2. How many hundredths of any principal equals the interest for 1 yr. at 6%? for 12 mo. at 6%?

What part of any principal equals the interest at 6% for 2 mo.? Why?

$\frac{1}{100}$ of any principal equals the interest for how long a time at 6%?

$\frac{1}{100}$ of $76 equals the interest of $76 at 6% for how long a time?

$.76 equals the interest of $76 for 2 mo. at 6%.

3. Write ten sentences similar to this:

$\frac{4}{100}$ of $784 equals the interest of $784 at 6% for 8 mo.

4. What is the interest of $76 at 6% for 9 mo.?

What equals the interest at 6% for 2 mo.? for 9 mo.?

$\frac{9 \cdot \$.76}{2} = ?$ $\frac{9}{2}$ of $\frac{1}{100}$ of $76 equals the interest of $76 for 9 mo. at 6%.

5. Make many sentences like this: $\frac{1}{2}$ of $\frac{1}{100}$ of $84 equals the interest of $84, at 6% for 11 mo. $\frac{11 \cdot \$.84}{2} = ?$

What is the interest at 6% of —

6. $784 for 11 mo.?
7. $970 for 2 yr. 7 mo.?
8. $8604 for 1 yr. 3 mo.?
9. $126.96 for 3 yr. 6 mo.?
10. $2960 for $1\frac{1}{2}$ yr.?
11. $60.84 for 1 yr. 11 mo.?
12. $85.24 for 1 yr. 9 mo.?
13. $114 for 2 yr. 7 mo.?

Practice until you can find quickly and accurately the interest on small sums for short periods.

14. In each of the above what is the ratio of interest to principal? of principal to interest?

Ans. to the sixth: $\frac{11}{2}$ of $\frac{1}{100}$, or $\frac{11}{200}$, is the ratio of the interest to principal. $\frac{200}{11}$ is the ratio of principal to interest.

1. Assuming 30 da. to be a mo., how many days in .1 of a mo.?

2. How many tenths of a mo. in 6 da.? in 9 da.? in 15 da.? in 7 da.? in 16 da.? in 28 da.?

How many months or tenths or hundredths months in each of the following:

3. 1 yr. 5 mo. 7 da.?

1 yr. 5 mo. 7 da. = 17.23 mo., or $\frac{1723}{100}$ mo.

4. 5 mo. 12 da.?
5. 6 mo. 15 da.?
6. 1 yr. 6 mo. 6 da.?
7. 3 yr. 7 mo. 9 da.?
8. 2 yr. 5 mo. 17 da.?
9. 1 yr. 9 mo. 19 da.?

10. What is the interest of \$225 for 2 yr. 7 mo. 11 da. at 6%?

$\frac{31.36 \cdot \$2.25}{2} = ?$ What is the interest of \$225 at 6% for 1 mo.? What, then, is the interest for 31.36 mo.?

11. Make ten sentences similar to this: $\frac{7}{10}\frac{8}{}$ of $\frac{1}{2}$ of $\frac{1}{100}$ of \$84 equals the interest of \$84 for 7 mo. 18 da. at 6%.

$\frac{76 \cdot \$.84}{10 \cdot 2}$ = the interest of \$84 at 6% for 7 mo. 18 da.

[1] What is the interest at 6% of—

12. \$245 for 6 mo. 18 da.?
13. \$100 for 8 mo. 21 da.?
14. \$960 for 3 yr. 7 mo. 9 da.?
15. \$274 for 4 yr. 10 mo.?
16. \$180 for 1 yr. 9 mo. 15 da.?
17. \$967 for 1 yr. 4 mo. 17 da.?
18. \$156.50 for 2 yr. 11 da.?
19. \$980 for 3 yr. 9 mo. 13 da.?
20. \$63.80 for 3 yr. 4 mo. 18 da.?
21. \$460 for 2 yr. 6 mo. 19 da.?
22. \$875 for 1 yr. 17 da.?
23. \$897 for 1 yr. 1 mo. 4 da.?

1. A note draws interest at 6% per annum. What is the ratio of one year's interest to the principal? of the principal to one year's interest?

2. What are these ratios at the end of 2 yr.? 3 yr.? 2 yr. 6 mo.? 1 yr. 4 mo.? 1 yr. 10 mo.? 2 yr. 2 mo.? 3 yr. 8 mo.? 2 yr. 3 mo.?

Ans. to last. The interest for 2 mo. equals $\frac{1}{100}$ of principal; the interest for 2 yr. 3 mo. equals $\frac{23}{2}$ of $\frac{1}{100}$, or $\frac{23}{200}$ of principal; $\frac{200}{23}$ is the ratio of principal to interest.

[1] Find first what *equals* the interest, then what is the interest.

3. \$54 is the interest of how much for 2 yr. 3 mo. at 6%?

What is the ratio of the principal to the interest? of principal to the amount due? of interest to amount?

Ans. $\frac{100 \cdot 2}{27}$ is the ratio of the principal to the interest. $\frac{200}{227}$ is the ratio of the principal to the amount. $\frac{27}{227}$ is the ratio of interest to amount.

4. \$9761 is the amount due on a note that has been drawing interest for 2 yr. 3 mo. at 6%. How much is the principal? How much is the interest?

5. What is the ratio of the interest to the principal for 8 mo. 12 da. at 6%? of principal to interest? of principal to amount?

6. What is the ratio of interest to the principal for 7 mo. 9 da. at 6%? of principal to interest? of principal to amount?

7. What is the ratio of interest to the principal for 4 mo. 8 da. at 6%? of principal to interest? of principal to amount?

8. \$612.81 is the amount due on a note that has drawn interest for 4 mo. 8 da. at 6%. What is the face of the note? the interest?

9. \$17.50 is the interest of \$500 for how long a time at 6%?

How much is the interest for 2 mo.? for 1 mo.? What is the ratio of \$17.50 to the interest for 1 mo.? What, then, is the ratio of the time to 1 mo.? The time is how many months?

10. \$17 is the interest of \$500 for how long at 6%?

What is the ratio of \$17 to the interest for 1 mo.? 6.8 mo. equals 6 mo. and how many days?

11. $31.20 is the interest of $400 for how long at 6% ? for how long at 8% ?

12. What is the interest of $\$x$ for y mo. at 6% ?

13. $\$a$ is the interest of how much for 11 mo. at 6% ? for b mo. ?

14. $\$c$ is the amount due on a note for 1 yr. 3 mo. at 6% for how much ?

15. $\$d$ is the amount due for m mo. at 6%. What equals the principal ? the interest ?

1. What is the ratio of 5% to 6% ? of 8% to 6% ? of 9% to 6% ? of 1½% to 6% ? of 7½% to 6% ? of 4½% to 6% ? of 8½% to 6% ? of 10% to 6% ? of 4% to 6% ? of 7% to 6% ?

Practice until these ratios can be thought readily.

2. What is the ratio of the interest of any principal at 8% to the interest for the same time at 6% ?

3. What is the ratio of the interest of any principal at 7½% to the interest for the same time at 6% ?

4. If $\$x$ is the interest at 6%, what is the interest for the same time at 8% ? at 4% ? at 7½% ? at 9% ?

5. What is the interest of $242 at 9% for 7 mo. ?

$\frac{3 \cdot 7 \cdot \$2.42}{2 \cdot 2} = ?$ What equals the interest of $242 at 6% for 7 mo. ? What, then, equals the interest at 9% ?

6. Make sentences similar to this :

$\frac{5 \cdot 13.2 \cdot \$7.84}{4 \cdot 2} =$ the interest of $784 for 1 yr. 1 mo. 6 da. at 7½%.

Find the interest of —

7. $360.75 for 3 yr. 3 mo. at 7%.
8. $247.50 for 2 yr. 11 mo. at 8%.
9. $843 for 11 mo. 27 da. at 7½%.
10. $1055 for 10 mo. 27 da. at 7½%.

11. $324 for 2 yr. 3 mo. 6 da. at 4½%.

12. $675 for 11 mo. 9 da. at 8%; at 9%; at 7½%; 10%.

13. What is the amount of $840, at 8%, for 6 mo. and 24 da.?

$$\frac{4 \cdot 6.8 \cdot \$8.40}{3 \cdot 2} + \$840 = \text{the amount.}$$

Find the amount of —

1. $275 at 6% from May 2, 1882, to June 1, 1883.
2. $375 at 7% from Dec. 14, 1880, to May 5, 1881.
3. $242 at 7½% from Aug. 27, 1872, to July 12, 1880.
4. $85684 at 9% from Aug. 17, 1881, to April 4, 1882.
5. $783 at 5½% from Sept. 19, 1868, to June 1, 1870.
6. $3248 at 6% from Jan. 4, 1882, to March 1, 1882.
7. $934 at 10% from Jan. 30, 1862, to Oct. 1, 1869.
8. $367.25 at 8½% from March 5, 1881, to Sept. 9, 1882.
9. $79.42 at 6½% from Jan. 1, 1873, to Sept. 5, 1873.
10. $847.42 at 7% from May 5, 1881, to Aug. 11, 1881.
11. Solve the above problems again more quickly.
12. Compare the interest of $3 for 1 yr. with the interest of $3 for 6 mo.; for 3 mo.; for 1 mo.
13. How much money will earn in 1 mo. as much interest as $500 earns in 1 yr.?
14. How much money must a man have on interest at 6% to have an income from it of $10 a da., counting 360 da. to the year?
15. How much must I invest in 4% bonds to receive from them an income of $1000 a year?
16. How soon will interest at 6% equal the principal? at 5%? at 7½%?
17. I had $1000. Loaned ⅛ of it at 4% for 2 yr. and the remainder at 5% for the same time. How much interest did I receive?

18. Loaned \$500 at 6% for 1 yr. 6 mo. How much more interest would I have received at 8% ?

19. Mr. B had \$800. He loaned $\frac{3}{4}$ of it for a year at 6% and the balance for the same time at 4%. What was the amount due ?

20. What is the ratio of the interest on \$300 for 4 yr. to the interest on \$600 for half as long ?

21. What ratio does the interest on \$300 for 1 yr. bear to the interest on twice the principal for double the time, the rate being the same ?

22. What principal will earn \$50 in 1 yr. 4 mo. at 6% ?

$\frac{100 \cdot \$50}{8} = \$625.$

$\frac{100}{8}$ is the ratio of the principal to the interest.

23. What principal loaned for 2 yr. 4 mo. at 5% will bear \$75 interest ?

$\frac{100 \cdot 6 \cdot \$75}{14 \cdot 5} = ?$

$\frac{6}{7}$ of \$75 equals interest at what % ?

$\frac{100}{14}$ is the ratio of principal to interest at what % ?

24. The interest for $1\frac{1}{4}$ yr. at 4% is \$100. Find the principal.

25. The interest for 4 yr. at 8% is \$400. Find the principal.

26. What principal will bear \$180 interest in $3\frac{1}{2}$ yr. at 7% ? At 6% the principal would yield what interest ?

1. At 6% per annum, what part of the principal equals the interest for 2 yr. 7 mo. 12 da. ? at 8% ? at $4\frac{1}{2}$% ?

.12
.035
.002
.157

What part of the principal equals the interest for 2 yr. ?

What part of the principal equals the interest for 7 mo. ? *Ans.* $\frac{7}{12}$ of $\frac{1}{100} = \frac{7}{200} = \frac{35}{1000}$.

What part of the principal equals the interest for 12 da. ?

Ans. $\frac{1}{10}$ of $\frac{1}{2}$ of $\frac{1}{100} = \frac{1}{2000} = \frac{5}{10000}$.

At 6% per annum, what part of the principal equals the interest for —

2. 2 yr. 4 mo.?
3. 1 yr. 7 mo.?
4. 5 yr. 8 mo.?
5. 2 yr. 9 mo.?
6. 3 yr. 11 mo.?
7. 7 mo. 7 da.?
8. 9 mo. 7 da.?
9. 5 mo. 21 da.?
10. 11 mo. 14 da.?
11. 5 mo. 29 da.?
12. 1 yr. 6 mo. 6 da.?
13. 2 yr. 7 mo. 9 da.?
14. 1 yr. 11 mo. 19 da.?
15. 4 yr. 4 mo. 4 da.?

16. At 6% per annum, what part of $840 equals the interest for 2 yr. 7 mo.?

17. How much is the interest on $840 for 1 yr. 7 mo. at 6%?

Exact Interest. — To find the exact interest for a part of a year, reckon 365 days to the year.

1. The interest of any principal for 52 da. equals how many 365ths of the interest for 1 yr.?

2. What is the exact interest of $750 for 70 da. at 7%?

$\frac{70 \cdot 7 \cdot \$7.50}{365}$ = the exact interest of $750 at 7% for 70 da.

Find the exact interest of the following:

3. $245 for 40 da. at 8%.
4. $95 for 75 da. at 6%.
5. $840 for 93 da. at 9%.
6. $743 for 45 da. at $7\frac{1}{2}$%.
7. $366 for 63 da. at 6%.
8. $244 for 33 da. at 8%.
9. $727 for 85 da. at 10%.
10. What is the exact interest on $1000 from March 1, 1885, to April 19, 1886, at 4%?
11. What is the exact interest of a note for $95, dated Jan. 10, 1891, due July 1, 1891?

Compound Interest. — 1. What is the compound interest of \$240 for 2 yr. 6 mo. at 6%?

What is the amount of \$240 for 1 yr. at 6%?

What is the amount of \$254.40 for 1 yr. at 6%?

What is the amount of \$269.66 for 6 mo. at 6%?

What is the difference between \$240 and \$269.66?

The difference between the last amount and the first principal is the compound interest of \$240 for 2 yr. 6 mo. at 6%.

State what you did in finding the compound interest of \$240 for 2 yr. 6 mo. at 6%.

What is compound interest?

How is compound interest found?

2. What is the amount of \$350 for 5 yr. at 4% compounded annually?

3. What is the compound interest of \$570 for 2 yr. 8 mo. at 10%?

4. What is the amount of \$600 at 8% for $1\frac{1}{2}$ yr. compounded semi-annually? How many interest periods are there? If 8% is the rate for 1 yr., what is the rate for 6 mo.?

5. What is the compound interest of \$92 at 7% for 2 yr. 4 mo. 6 da. compounded semi-annually? How many whole intervals of time in the above problem? How many months added in the fractional interval?

6. What part of \$1 equals the compound interest of \$1 for 3 yr. at 8%?

What part, then, of \$327 equals the compound interest of \$327 for 3 yr.?

7. What is the amount at compound interest of \$1 for 4 yr. 6 mo. at 7%? What is the amount of \$675 for the same time and rate?

8. What is the amount at compound interest of \$675 for 4 yr. 6 mo. at 4%?

Find the amount of each of the following at compound interest:

9. $745 for 3 yr. at 6%.
10. $1020 for 4 yr. at 6%.
11. $28.45 for $3\frac{1}{2}$ yr. at 7%.
12. $100 for 3 yr. 8 mo. at 8%.
13. $520 for 2 yr. 9 mo. at 8%.
14. $740 for 4 yr. 6 mo. at 7%.
15. If the compound interest, rate, and time are given, how find the principal?

Annual Interest. — If a note contains the words "with annual interest," or "interest payable annually," the interest is due at the close of each year, and, if not paid when due, this "annual interest" draws *simple interest* until paid.

1. Find the amount of $500 for 4 yr. 6 mo. with interest payable annually at 8%, interest unpaid.

What interest is due on the principal annually?

What is the total annual interest?

For what time does the first annual interest draw interest? the second? the third? the fourth?

The interest of $40 for 3 yr. 6 mo., 2 yr. 6 mo., 1 yr. 6 mo. and for 6 mo. is equal to the interest of $40 for what time?

The interest of $40 for 8 yr. at 8% is how much?

What is the amount of the annual interest, interest of the annual interest, and the principal?

.08 of $500 = $40. Interest due annually.

$4\frac{1}{2} \cdot \$40 = \$180$. Total annual interest.

8 · .08 of $40 = $25.60. Simple interest of $40 for 8 yr.

$500
180
25.60
$705.60 Amount due.

2. At the end of 3 yr., what is due on a debt of $400 with interest annually at 7%?

3. In what respect does annual interest differ from compound interest?

Partial Payments. — 1. The following is a promissory note:

$200. CHICAGO, June 4, 1889.

Two months after date, I promise to pay James Bergh two hundred dollars for value received, interest 6%.

HENRY PALLAS.

(*a*) Who is the maker of the above note?

(*b*) To whom is the note made payable, or who is the payee?

(*c*) How much does the maker, or drawer of the note promise, or what is the face of the note?

(*d*) Who is the holder or owner of the note?

(*e*) What is its date?

(*f*) When is the note to be paid?

(*g*) Tell as many things as you can about the note.

(*h*) What is a promissory note?

(*i*) What is the face of the note?

(*a*)

$600. ENGLEWOOD, ILL., June 28, 1888.

On demand I promise to pay to William Jones six hundred dollars with interest at 7%. Value received.

EDWARD DALE.

(*b*)

$600. ENGLEWOOD, ILL., June 28, 1888.

One year after date, I promise to pay William Jones, or order, six hundred dollars with interest at 7%. Value received.

EDWARD DALE.

(*c*)

$600. ENGLEWOOD, ILL., June 28, 1888.

One year after date, I promise to pay to William Jones or bearer six hundred dollars with interest at 7%. Value received.

EDWARD DALE.

(*a*) In what respect are the notes *a*, *b*, and *c* alike?

(*b*) How does note *a* differ from note *b*?

(*c*) How soon after date is note *a* payable? note *b*?

(*d*) To whom is note *a* payable? note *b*? Is note *b* payable to any one else?

(*e*) How may note *b* be transferred to some one else?

(*f*) If William Jones should write on the back of note *b*,

"Pay to John Condit,

William Jones,"

to whom would the note be payable?

(*g*) How could William Jones make the note payable to any one?

(*h*) If William Jones should write on the back of note *b*,

"Pay to John Condit or order,"

to whom could John Condit transfer the note?

(*i*) How does note *c* differ from note *b*?

(*j*) Explain the essential differences in the three notes.

(*k*) Why should "value received" be written in each note?

(*l*) Draw a demand note. Draw a time note payable to order; another to bearer.

When a part of a debt is paid, the amount and date of the payment are written on the back of the note. This is called an indorsement. What is an indorsement?

The following is the United States method of finding amount due upon a note on which partial payments have been made. Find the amount of the given principal to a

time when a payment or the sum of the payments is equal to or greater than the interest then due. Deduct the payment or the sum of the payments from the amount. With the remainder as a new principal proceed as before.

1. $200. AUBURN PARK, April 14, 1872.

On demand, for value received, I promise to pay to Ford McKnight, or order, two hundred dollars with interest at 6%.

JAMES HUGHES.

Partial payments were indorsed on this note as follows: Oct. 14, 1872, $46; April 14, 1874, $10; June 14, 1875, $50.

What was the amount due Jan. 26, 1877?

First principal,	$200.00
Interest to Oct. 14, 1872, 6 mo.,	6.00
Amount,	$206.00
First payment,	46.00
Second principal,	$160.00

The payment ($10) made April 14, 1874, is less than the interest ($14.40) due at that date; hence the interest is reckoned to June 14, 1875.

Interest to June 14, 1875, 2 yr. 8 mo.,	$25.60
Amount,	$185.60
Sum of second and third payments,	$60.00
Third principal,	$125.60
Interest to Jan. 26, 1877, 1 yr. 7 mo. 12 da.,	12.18
Amount due Jan. 26, 1877,	$137.78

2. A note of $500 is dated March 10, 1873.

Indorsements:

Nov. 10, 1873, $75; July 10, 1874, $100.

What was due May 10, 1876, at 6% interest?

3. A note of $100 dated Jan. 9, 1880.

Indorsements:

Oct. 15, 1881, $250; Dec. 27, 1883, $25; June 18, 1884, $350.

What was due July 13, 1885, at 6% interest?

4. A note of $700 dated March 12, 1883, is indorsed as follows:

June 10, 1883, $45; July 27, 1885, $24; Oct. 15, 1886, $200; Nov. 13, 1887, $190.

What was due Oct. 9, 1888, interest 8%?

Notes and accounts running for one year or less are often settled by what is known as the Mercantile Method.

The method is as follows:

Find the amount of the note or other obligation at the time of the settlement, and of each payment from the time it was made until the date of settlement.

From the amount of the note or other obligation subtract the sum of the amounts of the payments.

5. Solve the last two problems by the Mercantile Method. Which method is more favorable to the creditor? Which seems more just? Why?

6. A note of $850 dated Feb. 6, 1870, and drawing interest at 7½%, was indorsed as follows:

May 15, 1870, $270; Sept. 12, 1870, $80; Nov. 20, 1870, $120.

What was due Jan. 1, 1871?

7. On a note for $740 at 10% interest dated Feb. 1, 1887, and maturing 10 months after date, the following indorsements were made:

April 15, 1887, $60; May 3, 1887, $85; Sept. 12, 1887, $300.

What was the balance due at the time of payment?

Present Worth. — 1. If x is the amount of money a farmer had after a gain of $\frac{1}{5}$, what equals the sum on which he gained the $\frac{1}{5}$?

2. What is a man worth if \$535 is .08 more than what he is now worth?

3. If the future value of a lot is \$990, which is 10% more than its present worth, what part of \$990 equals the present worth?

4. If \$$x$ were the amount of money a man agreed to pay you at the end of a year, and money were worth 20% to you, what would you be willing to take for the debt to-day?

If you add $\frac{1}{5}$ or 20% to what you are willing to take, does the sum equal \$$x$?

What you are willing to take equals how many sixths of \$$x$?

5. How much cash is equivalent to a claim of \$224 payable in 2 yr., provided the use of the money is worth 6% per annum?

\$200 is the present worth of the above debt; what is the present worth of any debt payable at a future time?

\$24 is the true discount; what is the true discount of any debt?

6. What ought I to receive for a note of \$275, which bears no interest, and which will become due in 2 yr. 4 mo., the use of the money being worth 9% per annum?

$\frac{100 \cdot \$275}{121} = ?$

7. What is the difference between the true discount and the interest of \$317 for 3 yr. at 7%?

8. What is the face of a note due in 6 mo., with interest at 8%, that will cancel a debt of \$232 due in 6 mo. without interest?

9. What is the difference in value between a cash payment of \$225 and a note of \$230 due in 8 mo. without interest, the use of money being worth $7\frac{1}{2}\%$? if the use of money is worth 5% ?

Borrowing Money. — The systems of loaning money at a bank and by private individuals are not alike.

If A borrows \$300 of B for 1 year, at 10%, when does A pay the interest? What amount of money does A pay B at the end of the year?

If A wishes to borrow about \$300 for 2 mo. at a bank, he makes a note for \$300, *without interest*, and presents it, properly secured. The banker computes the interest on the face of the note for 63 da., and deducts this amount from the *face*, paying A the difference.

If A holds an interest-bearing note against B for \$300 due in 1 yr., at 10%, and wishes to obtain money by selling that note at a bank, he presents it, properly secured. The banker deducts interest for 3 da. more than the specified time from the *amount* of the note (\$330) instead of from its face.

The amount a banker deducts is called *bank discount*. It equals the interest, as just described, of the amount that he can collect on the note for three days more than the time specified.

The sum paid after deducting the bank discount is called *proceeds*.

The three days more than the specified time are called *days of grace*.

A note *matures*, or is legally due, on the last day of grace. If the last day falls on Sunday or a legal holiday, it is due the day before.

In what ways do the systems of borrowing money at a bank and of a private individual differ?

Suppose A gives a bank a note for $300 and receives $294 as the proceeds; how much does the bank reserve as interest?

The $300 is the sum of the proceeds and what?

The $6 is the interest on the proceeds and on what else?

What does a bank gain by deducting, at the time the loan is made, interest on the face or the amount of a note?

In addition to obtaining interest on interest, what other advantage does the bank gain?

When A borrows of B, who has the use of the interest until the debt is paid? When A borrows of a bank?

What does a bank receive at the date of maturity of a non-interest-bearing note? On an interest-bearing note?

On what is the bank discount reckoned in dealing with a non-interest-bearing note? On what, then, should it be reckoned in dealing with an interest-bearing note?

Does a man who borrows money at a bank pay interest for the three days of grace?

What does the debtor gain, if anything, by being allowed three days of grace?

Explain the system of borrowing money at a bank on an interest-bearing and on a non-interest-bearing note.

What does a bank gain by deducting interest on the face or amount of the note at the time the loan is made?

1. What is the bank discount of a note of $150 payable in 60 da., discounted at 8%? What are the proceeds?

What part of the face of the note equals the discount for 63 da.? $\frac{4 \cdot 11 \cdot \$1.50}{3 \cdot 10} = \text{bank discount.}$

Find the bank discount and the proceeds of a note of —

2. $100 payable in 60 da., discounted at 6%.
3. $75 payable in 90 da., discounted at 10%.
4. $25.50 payable in 30 da., discounted at 8%.
5. $727 payable in 100 da., discounted at 10%.

6. $50 dated June 1, 1875, payable Sept. 1, 1875, discounted at 10%.

What is the actual time in days between June 1 and Sept. 1?

7. $170 dated March 7, 1887, payable July 9, 1887, discounted at 9%.

8. $11.75 dated Jan. 1, 1889, payable March 1, 1889, discounted at 9%.

9. What is the difference between the true discount and the bank discount of $874 due in 100 da., discounted at 6%?

10. What is the difference between the true discount and the bank discount of $95, due in 57 da., discounted at 8%?

11. What is the date of maturity of a note, given May 10, 1884, and payable in 90 da.?

Note. — To find the date of maturity when the time is expressed in days, count forward for three days more than the given number of days. When the time is expressed in a certain number of months, the note is nominally due on the corresponding day of the month, or, when there is no corresponding day, on the last day of the month.

A 2-mo. note dated Dec. 31, 1888, is due Feb. 28. When is a 3-mo. note dated Dec. 31, 1888, due?

When is a 2-mo. note dated July 31, 1857, due?

12. A note, dated Jan. 7, 1885, was payable in 60 da.; what was the date of maturity? If the note had been payable in 2 mo., what would have been the date of maturity?

13. If $1 were the proceeds of a note out of which .1 had been taken, $1 would equal what part of the face of the note?

14. If $250 were the proceeds of a note out of which .06 had been taken, $250 would equal how many hundredths of the face of the note?

15. If m dollars were the proceeds of a note out of which .03 had been taken, m dollars would equal what part of the face of the note?

16. If $450 were the proceeds of a note which had been discounted 6%, what would be the face of the note?

17. For what sum must a note, payable in 57 da., be drawn to produce $95 when discounted at 6%?

18. What is the face of a 90-day note which, discounted at 7%, will give as proceeds $275?

1. What is the cost of a sight draft on Chicago for $485 at 1% premium?

2. What is the cost of a draft for $2840, discount $\frac{3}{8}$%?

Find the cost of drafts for the following amounts:

3. $564, premium $\frac{1}{8}$%.
4. $2700, premium $\frac{1}{2}$%.
5. $6000, premium $\frac{3}{4}$%.
6. $475, discount $\frac{5}{8}$%.
7. $185, discount $1\frac{1}{2}$%.
8. $837, discount $\frac{3}{8}$%.

9. If the cost of a sight draft at $\frac{3}{4}$% premium is $873, what is the face of the draft?

Find the face of a draft which cost —

10. $620, premium $1\frac{1}{2}$%.
11. $396, premium $\frac{3}{8}$%.
12. $2400, discount $\frac{1}{2}$%.
13. $847, discount $\frac{3}{8}$%.

14. Find the cost of a draft of $560 payable in 4 mo., interest at 7% allowed until the draft is paid, premium $\frac{3}{4}$%.

What would the draft cost if no deduction was made for the interest?

What is the interest of $560 at 7% for 4 mo.?

$564.20 less the interest equals what?

Find the cost of drafts for the following amounts:

15. $285, premium $1\frac{1}{8}$%, time 60 da., interest 4%.
16. $1000, premium $\frac{3}{8}$%, time 2 mo., interest 6%.
17. $2840, premium $\frac{1}{2}$%, time 90 da., interest 5%.
18. $700, discount $\frac{3}{4}$%, time 3 mo., interest $4\frac{1}{2}$%.
19. What is the face of a draft bought in New York,

which will pay a debt in Chicago of $2500, exchange on New York being $\frac{1}{2}\%$ premium in Chicago?

20. A commission merchant in Chicago pays by draft a merchant in Kansas City a debt of $480, drafts on Chicago being at a premium of $\frac{3}{4}\%$ in Kansas City. What is the face of the draft which pays the debt?

21. An agent in Davenport sold a consignment of goods for $2856, commission at the rate of 2%. He remitted the proceeds by draft on Chicago at a premium of $\frac{3}{8}\%$. What was the amount remitted?

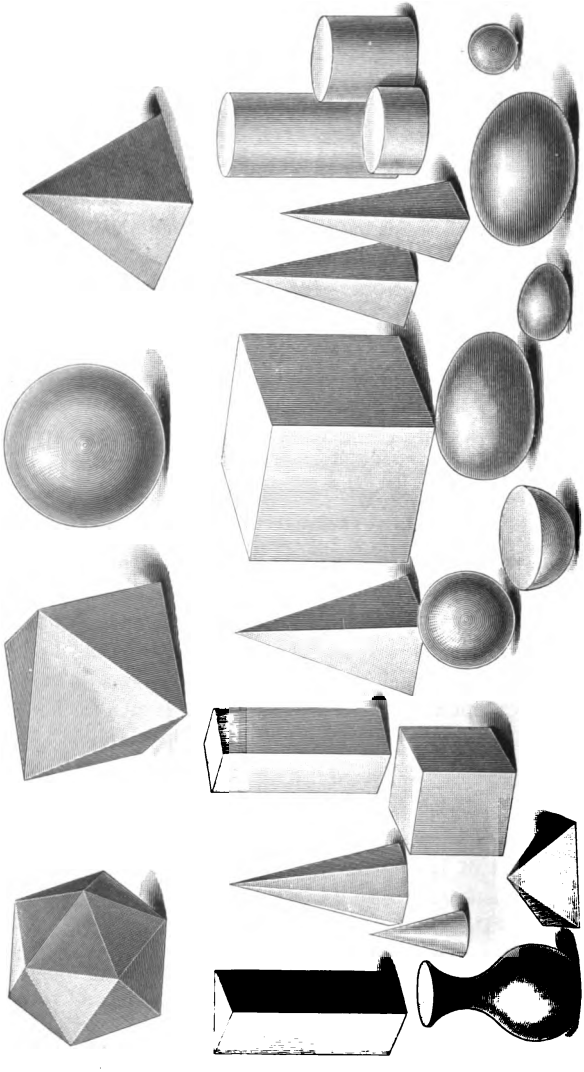
MENSURATION.

"Geometry reduces its laws and attributes of magnitude to perfect clearness by according to the senses a representation of those lines, surfaces, and solids which it conceives with the utmost completeness and precision; and thus, issuing forth from behind the veil of mental invisibility into the visible and palpable, its doctrines may almost be seen and handled, and yet without losing aught of their purity and necessity. Thus, geometry, if I may so express myself, becomes a thinking with the eye. . . . This relation of its laws to determinate figures; this apprehension of the highest and most surprising doctrines through the visibility of body is precisely what at once attracts and animates the young — what gradually elevates and prepares for high abstraction their powers as yet incapable of such exercise." — THIERSCH.

"Unless the definitions are intuitions of the figures and relations defined, they are also barren. . . . The mathematician does not begin by assuming the properties of figures, and after defining them proceed to ascertain whether such figures exist; he begins by ascertaining that such figures and such relations do exist, and then defines them as he finds them. In other words, definitions are the expressions of the figures, not their foundations." — G. H. LEWES.

"So far as the student has the power let him seize the most general point of view within his reach, . . . but let him shun the common fault of taking indistinctness as evidence of generalization, for this is to suppose ourselves on a mountain because we are in a mist." — Dr. WHEWELL.

"Gauss, whose authority on such a subject weighs against a whole academy, declared geometry to be the 'science of the eye'; and Professor Sylvester, also a very considerable authority, declares that most if not all the great ideas of modern mathematics



SOLIDS.

had their origin in observation. . . . Even in the higher developments of the Calculus, where sensible experiences seem most widely departed from, it is easy to trace a sensible origin for the extra sensible intuitions. . . . Most of the difficulties in this science are difficulties rather of intuition than of reasoning."—GEO. H. LEWES.

"The higher processes of mind in mathematics lie at the very foundation of the subject."—JOSEPH SYLVESTER.

Solids, Surfaces, and Lines. — 1. In how many directions can you extend your hand?

In how many directions does a sphere extend from any point within it? an ovoid? a prism? a cylinder? a lump of coal? a cone? a cube?

In how many directions does the air of this room extend from any point within?

2. How far can we think extension in any one direction? How far can we think extension in every direction? Of what can you think that is limited in its extent in every direction?

Name a solid, that is, something whose extension is limited in every direction.

Think whether a cylinder, an orange, the air in the room, the water in a tumbler can be called solids.

Give other examples of portions of space limited in every direction.

3. What limits the extension of an apple? of a sphere? of a pencil? of a brick? What is a surface?

Here is a glass partly full of water. What is in contact with the upper surface of the water? Has the air in the glass a surface as well as the water? Has the air one surface and the water another? What limits or bounds the extension of the water? Then what is the boundary of the water? What limits the extension of the air in this room? Then what is the boundary of this body of air?

What limits the extension of a sphere ? of a cube ? of any solid ?

Of how many smaller surfaces is the entire surface of a cube composed ? Show me the boundary of one of these smaller surfaces.

The surface of a hemisphere is composed of how many smaller surfaces ? What limits the extension of the plane surface ? of the curved surface ?

Move your hand along the edge or boundary of the top of your desk; along the boundary of the upper surface of the table.

Think of a square piece of paper. Think of the edges or limits of its extension. Think of the edge or boundary of the surfaces of different objects.

What is an edge ?

4. Place a finger on a corner of the cube. In how many directions from one of its corners do the edges of a cube extend ?

Think of the *extension* of each of these edges.

Find the upper right corner in the front of the room. In how many directions from this corner do the edges of the room extend ?

In how many directions from the corner of the blackboard do its edges extend ? Think of the extension in *one direction* of an edge of a cube. Of what kind of a line did you think ?

Think of extension in one direction.

What is a straight line ?

What kind of a line do you think when your mind moves along an edge of a cube ? of a pane of glass ?

Is there any difference between an edge and a line ?

Name things that represent or suggest straight lines, or extension in one direction.

5. What is found at each end of one of the edges of a

cube? How many lines meet or end at this point? Does a point have extension?

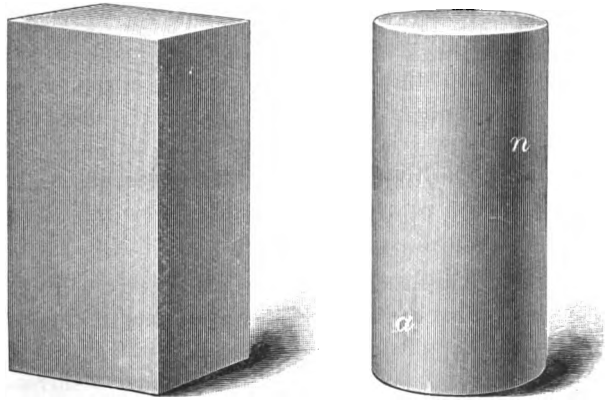
What is a point?

A point is position.

What is used to represent a point?

Note.— Pupils should be able to state questions before trying to answer them.

6. Observe the curved surface of a cylinder. Can two points be selected in the curved surface of a cylinder



that cannot be connected by a straight line lying wholly within this surface?

If the mind thinks one unchanged direction in passing from *a* to *n*, does the line *an* lie within the surface of the cylinder?

Can two points be so placed in the curved surface of the cylinder that they may be connected by a straight line lying wholly in the surface? Can any two points whatsoever in the surface of the blackboard be connected by a straight line lying wholly in its surface? Try to find two that cannot be so connected.

Are there any two points in the top of this desk that cannot be so connected?

The blackboard is a plane, so is the top of this desk. How does a plane differ from a curved surface?

What is a plane?

A plane is a surface any two points of which may be joined by a straight line lying wholly within the surface.

7. What is extension in every direction? What is extension in one direction? What is a solid?

What is the boundary of a solid? The beginning and the limit of extension in one direction are called what?

Make a drawing to represent extension in one direction. What is a straight line? Why do not the edges of a cylinder suggest straight lines? Do the edges of a cube? Why? Represent a point.

What is a point? Why is the intersection of two lines a point? Does a point have extension?

What is a plane?

Relations of Magnitude.[1] — Find equal solids. Find unequal solids. Find equal surfaces. Find unequal surfaces. Find equal lines; unequal lines.

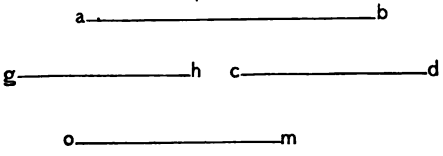
What relations of magnitude may solids have? surfaces? lines?

Relations of Direction. — 1. Find lines extending in the same direction and lines extending in different directions. With respect to direction what relations may lines have?

What is true of the direction in which the right and left edges of the door extend? Can you think of them as extending in opposite directions? What is true of the upper and lower edges of the blackboard? Find other

[1] "When thinking of a thing's figure, we think of the relation of magnitude which its constituent parts bear to one another; but when thinking of its size, we think of the relation of magnitude which it, as a whole, bears to other wholes." — HERBERT SPENCER.

edges in the room that suggest the same direction; that suggest opposite directions. How many edges of a cube extend in the same direction? Show four edges of the room extending in the same direction. Try to find three groups of four edges of the room that extend in the same direction. Do so with a cube.

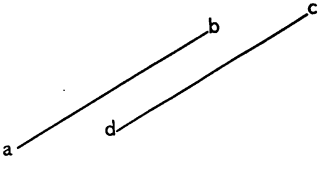


What is true of the direction in which the lines *ab*, *gh*, *cd*, and *om* extend?

Lines thought of as extending in the same direction are called *parallel lines.* Who has seen parallel roads, parallel surfaces, parallel sidewalks, people walking in parallel directions?

If the parallelism of lines consists in their sameness of direction, are Carl and Lyle traveling in parallel directions when one walks toward the east and the other toward the west? What must one of them do to make the directions parallel?

If parallel lines are lines which extend in the same direction, how many pairs of parallel lines are suggested by the edges of the blackboard?



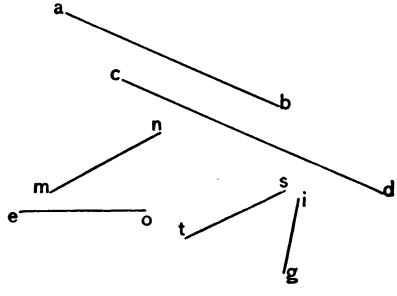
2. If *ab* represents one direction and *cd* an opposite direction, are the lines *ab* and *cd* parallel? Are the lines *ab* and *dc* parallel? Why? (Notice that the lines referred to in the last question extend from *a* to *b* and from *d* to *c*. The direction is indicated by the order in which the letters are read.)

Review.

Remark. — The perceptions necessary to make permanent possessions of the things perceived are not repetitions of words, but of ideas, and to get the thought again and again before the pupil is possible only by arousing the mind to activity.

What relations of magnitude may solids have? surfaces? lines? What relations may lines have with regard to direction? What are parallel lines? If a boy travels one rod east, and another one mile east, are the directions parallel?

Angles. — 1. What is true of the direction in which lines *ab* and *cd* extend? What is true of the directions in which the lines *mn* and *eo* extend? Draw two lines so that they make a slight difference in direction. Draw two others so that they make a greater difference in direction.



2. How must two lines extend to indicate a sameness in direction? to indicate a difference in direction?

3. Which is greater, the difference in direction of the lines *mn* and *eo*, or of the lines *ts* and *gi*?

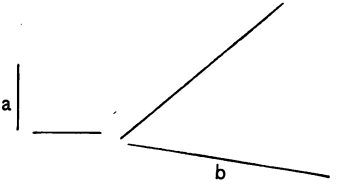
4. Find lines of a cube extending in different directions. Find lines in the room extending in different directions. Find walls of the room extending in the same direction; in different directions.

5. How many directions are required to make a difference in direction? To see a difference in direction is to see an angle. What is an angle? If an angle is a difference in direction, how many directions are required to make an angle?

1. How many lines are required to make a sameness in direction? to make a difference in direction? The difference in direction of two lines lying in the same plane is called a *plane angle.*

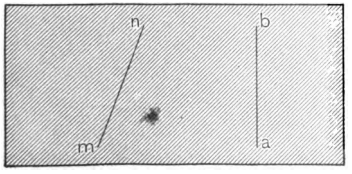
2. Can any two lines whatsoever drawn in the same plane and forming an angle be made to meet if they are extended? What is the exception?

3. Draw lines in opposite directions. What is an angle? Is there a difference in the direction of the lines just drawn? Do they form an angle? If these lines are thought of as extending in the same direction, what term is applied to them?



4. Do you see an angle in diagram *a*? in diagram *b*? Which is the greater angle? Why is *a* the greater angle? What makes one angle greater than another? Does it make an angle larger to lengthen the lines that form it? Can two short lines make as large an angle as two long lines? Show this. Can the lines in diagram *a* be produced so that they will meet? in diagram *b*?

5. Draw lines in opposite directions. What is an angle? Do the lines just drawn form an angle? Do lines thought of as extending in different directions form angles?

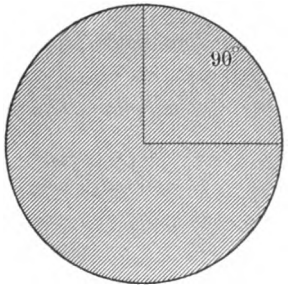


6. Can lines be drawn on the blackboard which, if produced, will never meet? Will the lines *ab* and *mn* meet if they are extended?

7. Can all lines drawn on the blackboard and forming angles be extended so that they will meet? What is the exception?

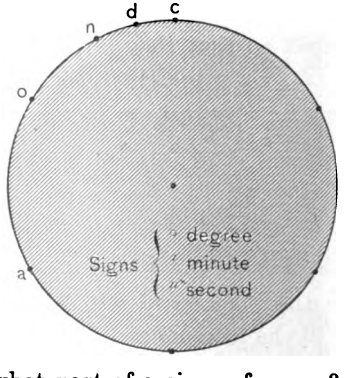
If the lines of an angle meet, the point of meeting is called the *vertex* of the angle.

1. Draw an angle. Describe a circle, using the vertex of the angle as the center. Estimate the part of the circumference of the circle that is intercepted by the lines of the angle. If the circumference of the circle is divided into 360 equal arcs, or degrees, how many of these degrees are intercepted by the lines of the angle? Then how many degrees measure the difference in direction of the lines of the angle?



2. Will the number of degrees be greater if the circle is larger? less if the circle is smaller? Draw five angles of different sizes. Estimate the number of degrees in each. Use a protractor and measure the angles to correct the estimate made.

3. Draw an angle. Draw a larger angle. Draw the largest angle that you can. In what directions do the lines of the largest angle extend?



1. Into how many arcs each equal to the arc *ao* can the circumference of this circle be separated?

2. The distance from *a* to *o* is 60°. How many 60° in the circumference of the circle? 60° equals what part of a circumference?

3. What is the distance from *o* to *n*? How many 30° in the circumference of this circle?

4. What is the distance from n to d? The arc ao equals how many 15°? The circumference of a circle equals how many 15°?

5. Draw an arc of 60°; of 30°; of 15°.

6. One degree equals what part of the circumference of a circle? 1° equals 60′. (*Read:* 1° equals 60 minutes of distance.) 60′ equals what part of the circumference of a circle? 60′ equals what part of the distance from n to d? from o to n?

7. The length of 1° on the equator is 60 geographical miles. What is the length of 1′ on the equator?

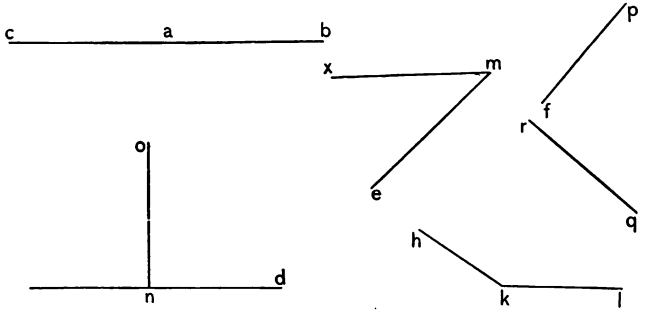
8. 1′ equals 60″ (″ seconds of distance). If 1 mile is the length of 1′ on the equator, what is the length of 1″ on the equator?

9. If the length of 1′ on the parallel which forms the north boundary of British Columbia is $\frac{1}{2}$ of a geographical mile, what is the length of 1° on the same parallel? of 30′? of 30″? of 1″? of 2°?

10. 1″ equals what part of 1′?

1′ equals what part of 1°?

1° equals what part of the circumference of a circle?



What is true of the direction in which the lines *ab* and *ac* extend? The difference in direction of the lines *ab* and *ac* is a *straight angle.*

Think of the difference in direction of lines extending in opposite directions.

What is a straight angle

If a straight angle is the difference in direction of two lines that extend in opposite directions, is the angle *ond* a straight angle? The angle *ond* equals what part of a straight angle? See p. 143.

The angle *ond* is a right angle.

A right angle equals what part of a straight angle?

What is a right angle?

Are the lines *fp* and *rq* perpendicular to each other?

The lines *fp* and *rq* form what kind of an angle?

The angle *xme* is an acute angle. The acute angle *xme* is less than one-half of what angle? It is less than what angle?

What is an acute angle?

Is an acute angle less or greater than a right angle?

The angle *hkl* is an obtuse angle.

What is an obtuse angle?

In what position are the lines of a right angle with regard to each other?

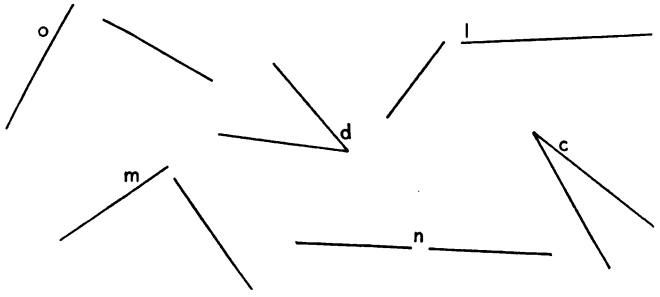
Is there any angle greater than a right angle which is not an obtuse angle? Which angle is it?

1. Draw a right angle having long lines and a right angle having short lines. Which, if either, is the greater? Why are they equal? What kinds of angles are always respectively equal?

2. What is true of the magnitude of obtuse angles? of acute angles? Are obtuse angles always equal? acute angles?

Note. — Acute and obtuse angles are also called oblique angles, and the lines which form them oblique lines.

3. Give names of the angles in the diagram. How many are right? straight? acute? obtuse? oblique?



4. Find two right angles. Find the two pairs of perpendicular lines.

5. Find the three oblique angles. How many of the oblique angles are acute?

6. Find three pairs of oblique lines. The oblique lines are the lines of which angles?

7. Find edges of solids forming right angles; acute; obtuse.

8. Do you know of any streets that form oblique angles?

9. How many angles are made by two intersecting streets or by two intersecting lines?

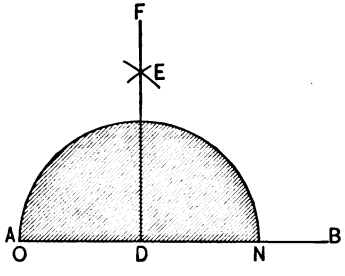
10. If one of the four angles formed by two intersecting lines is a right angle, what is true of the magnitude of the other three? if one is acute? if one is obtuse?

To Draw Perpendicular Lines. — It will aid greatly in making boxes, cubes, and other solids accurately and easily, to learn to draw perpendicular lines and to handle the ruler and dividers readily.

The pupils should first see the drawings as wholes. Afterward they may observe the teacher draw and learn a method of proceeding by seeing and doing, not by following step by step a formal

direction such as, "Let AB be a given line and D a point within it. From D as center," etc.

Growth in the power to express orally should increase as perceptions and power to express by hand increase. But there is danger of confusing the pupil by pushing the means into the foreground through verbal description. When he has a mental whole which he wishes to construct and sees the teacher make it many times easily and quickly, he readily gains similar constructive power, without dwelling on the mechanism.



Practice drawing perpendicular lines. Write a direction for drawing perpendicular lines.

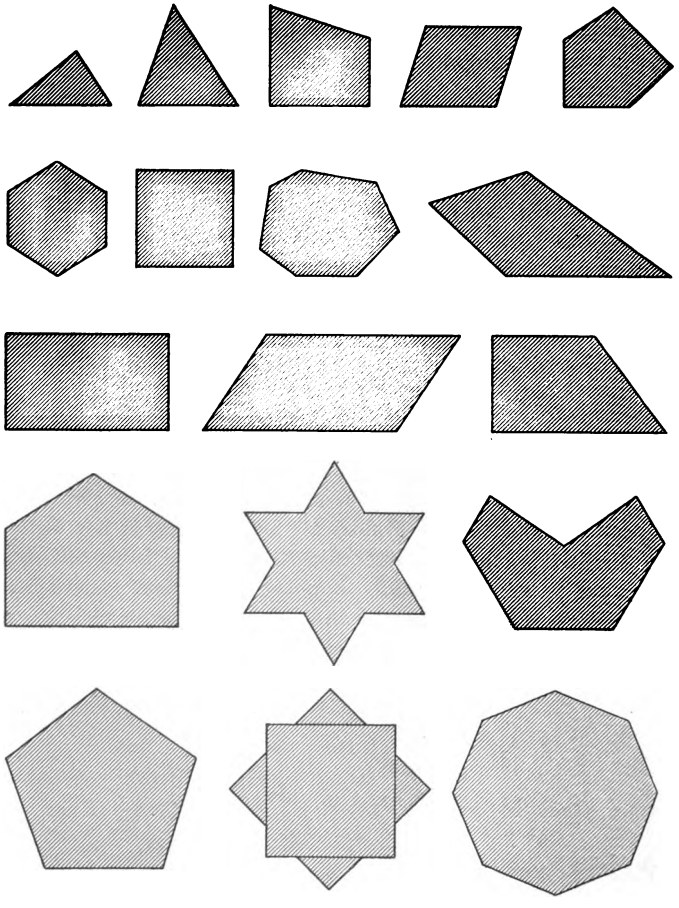
1. Can you draw right angles of different sizes? Try it.
2. Can you draw straight angles of different sizes? Try it.
3. How many acute angles of different sizes may be drawn?
4. How many obtuse angles of different sizes may be drawn?
5. What is the least possible number of acute angles that may form one right angle?
6. What is the least possible number of acute angles that may form an obtuse angle?
7. What is the least possible number of acute angles that may form a straight angle?
8. An obtuse angle and, at least, how many acute angles equal a straight angle?
9. The sum of a right angle and what equals an obtuse angle? What equals the difference between a right angle and an obtuse angle?

10. The sum of a right angle and what equals a straight angle? What equals the difference between a right angle and a straight angle?

Review. — What is space? What is a solid? What is the limit of a solid? What is a straight line? What is a point? What is a plane? What are parallel lines? What is an angle? What relations of magnitude may solids have? surfaces? lines? What relations of direction may lines have? What is a plane angle? What is the vertex of an angle? Define the different kinds of angles. What lines are perpendicular? What general name is given to acute and obtuse angles? What lines are oblique?

To Teacher. — The reviews are not to be a mere matter of question and answer. It is not expected that the student will at this stage grasp the full significance of these ideas; the developing mind must have time and increasing experience to give ideas expansion. But the opportunity for seeing things in new lights, for making the needful associations, should be freely afforded. Attempts to apply the adult standard will defeat growth into higher and more complete ideas. There can be no real review and no real progress without renewing interest from time to time, and thus gradually unifying and enlarging ideas.

1. What is a plane?
2. What is a polygon?
3. In what respect are the polygons alike?
4. What kind of a figure is a polygon? By what are polygons bounded?
5. What is a *polygon*? A *polygon* is a plane bounded by straight lines.
6. What is the least number of sides a polygon may have? the greatest number?
7. What is a *triangle*? A *triangle* is a polygon of three sides.



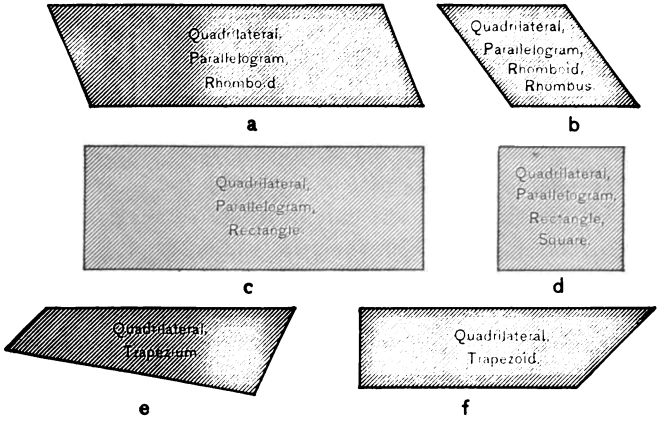
POLYGONS.

8. What is a *quadrilateral*? a *pentagon*? a *hexagon*? a *heptagon*? an *octagon*?

9. What does the word "*polygon*" mean?

10. Find polygons in the room and tell how many sides

they have. *Ex.* The surface of the blackboard is a polygon of four sides. The base of this hexagonal prism is a polygon of six sides.



Quadrilaterals. — 1. Draw quadrilaterals. Draw equilateral quadrilaterals. Draw an oblique-angled quadrilateral. Draw a right-angled one.

2. How many of these polygons are quadrilaterals? How many are equilaterals?

3. How many are right-angled? How many are oblique-angled?

4. In what are *a*, *b*, *c*, and *d* alike? What is true of the opposite sides of *a*, *b*, *c*, and *d*? What common name do *a*, *b*, *c*, and *d* have that *e* and *f* do not have?

5. How many of these polygons are parallelograms?

6. In what are *a* and *b* alike? In what do they differ?

7. What common name do *a* and *b* have that the other parallelograms do not have?

8. In what are *c* and *d* alike? In what do they differ? What common name do *c* and *d* have that *a* and *b* do not have?

9. Are all quadrilaterals parallelograms? Are all parallelograms quadrilaterals?

10. Are all squares rectangles? Are all rectangles squares?

11. Which quadrilateral has only one pair of parallel lines? Which has no parallel lines?

12. How many kinds or classes of quadrilaterals are there?

13. Cut two of each kind, making each pair as unlike as possible. In which pair is there the least variation in shape? Is there not one pair in which there is no variation in the angles nor in the relative length of the lines? Which is it? In one pair what is the only variation that can be made? Why is there less variation in the shape of the quadrilaterals having right angles than in those having oblique angles?

14. What is wrong with this definition of a square: A square is a right-angled parallelogram?

15. What is wrong with this: A square is an equilateral parallelogram? Make a definition of a square that will not include any other parallelogram.

16. Write in the fewest words possible such a description of each quadrilateral that the person reading it can select the quadrilateral described. *Ex.* Find or draw an equilateral oblique-angled parallelogram.

17. Is there more than one kind of quadrilateral that is both equilateral and oblique-angled?

18. [1] Write a definition of each of the following:

[1] "The axioms require not to be granted, but to be seen. If any one were to assent to them without seeing them to be true, his assent would be of no avail for purposes of reasoning. . . . Supposing we could deduce our reasoning from definitions alone, it must be allowed, I think, that still our geometrical propositions would properly depend, not on the definitions, but on the act of the mind by which we fix upon such definitions." — DR. WHEWELL.

A polygon, a quadrilateral, a parallelogram, a rectangle, a square, a rhomboid, a rhombus, a trapezoid, a trapezium.

Picture mentally the different quadrilaterals.

A *quadrilateral* is a polygon of four sides.

A *parallelogram* is a quadrilateral whose sides are parallel.

A *rectangle* is a right-angled parallelogram.

A *square* is an equilateral rectangle.

A *rhomboid* is an oblique-angled parallelogram.

A *rhombus* is an equilateral rhomboid.

A *trapezoid* is a quadrilateral having only one pair of parallel sides.

A *trapezium* is a quadrilateral having no sides parallel.

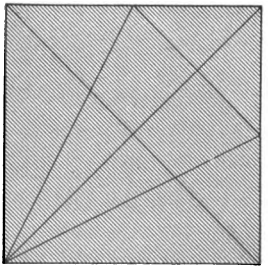
What is a square? a rectangle? a parallelogram? a quadrilateral? a polygon?

Review from the beginning.

Finding Forms made by Folding. — Give each pupil a four-inch square. Place it for folding.

Fold the square so that the right-hand point in front will coincide with the left-hand point at the back.

Crease. Open the paper. Fold the square so that the left-hand point in front will fall upon the right-hand point at the back.



Crease. Open the paper. Fold the paper so that the front edge will fall upon the diagonal which was creased first.

Crease. Fold the left-hand edge so that it will coincide with the same diagonal.

Crease. Fold the paper so that the right-hand point at the back will fall upon a diagonal, and so that an isosceles triangle will be formed.

Crease paper carefully. Unfold the paper so that you

will have the square again. Observe the forms made by the creased lines.

1. Observe the figure. How many triangles each having a right angle can you find?

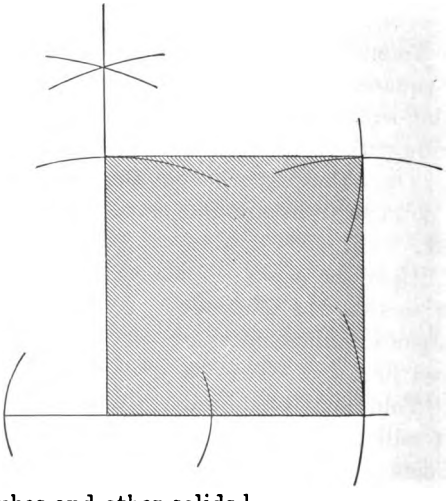
2. How many triangles having two sides equal can you find?

3. How many triangles can you find having no two sides equal?

4. How many trapezoids can you find?

5. How many different figures have you found in the square?

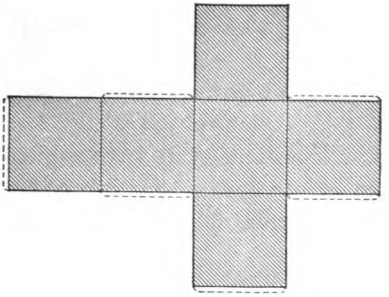
To Teacher. — Draw the square several times in presence of pupils. See note, page 145.



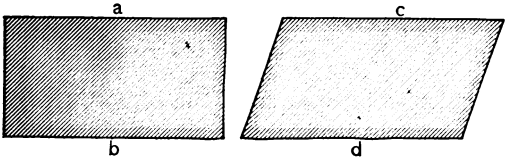
Make cubes and other solids.[1]

[1] Making some of the simple solids at this time prepares for the development of the more complex solids when the study of the solids is begun. The work of making the solids can be done at home. The

Suggestion. — Draw on thick, tough cardboard the diagram shown below. Cut the diagram out of the cardboard and cut half through the edges of the surfaces that are joined. Then fold into the solid. Fasten the adjacent surfaces of the solid by means of mucilage or paste.



Areas of Parallelograms. — Show the line on which each of the parallelograms rests and the line opposite, *i.e.*, show the bases of the parallelograms.



Show the bases of the blackboard.

Find other parallelograms and show their bases.

The altitude or height of a parallelogram is the perpendicular distance between what? What is true of the

first work handed in by the pupils will probably be soiled and poorly put together. Mistakes in the diagram will be manifest in imperfect solids. Continued practice is needed to give muscular control and the habit of ready and effective action. Such action is the product of growth. The particular thing made is of consequence only as it affects the condition, — the development of the individual. A display of finely finished forms is not necessarily evidence of growth.

comparative altitude of the parallelograms above? Draw oblique-angled parallelograms on the blackboard and measure their altitude.

Review pp. 279 to 282, "Elementary Arithmetic."

Areas of Rectangles.—1. Draw a rectangle 5 in. long and 3 in. wide.

What is the number of sq. in. in a rectangle 5 in. long and 1 in. wide?

What, then, is the number in a rectangle 5 in. long and 3 in. wide?

2. What is the area of a rectangle 5 in. long and $4\frac{1}{2}$ in. wide?

$\frac{9 \cdot 5}{2}$ = the number of sq. in. in the area.

What is the number of sq. in. in a rectangle 5 in. long and $\frac{1}{2}$ in. wide? What, then, equals the number in a rectangle 5 in. long and $\frac{9}{2}$ in. or $4\frac{1}{2}$ in. wide?

3. Find the area of a rectangle $6\frac{2}{3}$ ft. long and $3\frac{3}{7}$ ft. wide.

$\frac{24 \cdot 32}{7 \cdot 5}$ = the number of sq. ft. in the area.

What is the number of sq. ft. in a rectangle $6\frac{2}{3}$ ft. long and 1 ft. wide? What, then, equals the number of sq. ft. in a rectangle $6\frac{2}{3}$ ft. long and $\frac{1}{7}$ ft. wide? What, then, equals the number of sq. ft. in a rectangle $6\frac{2}{3}$ ft. long and $\frac{24}{7}$ or $3\frac{3}{7}$ ft. wide?

4. What is the number of sq. in. in the top of a writing table $2\frac{1}{8}$ ft. long and $1\frac{1}{2}$ ft. wide?

5. What is the area of a window pane whose width is $2\frac{2}{3}$ ft. and whose altitude is $2\frac{5}{12}$ ft.?

6. Find the number of sq. yd. in the floor of the schoolroom.

7. A garden is $8\frac{1}{2}$ rd. long and $5\frac{1}{4}$ rd. wide. What is its area?

8. How many sq. in. in a sheet of paper $8'' \times 13''$?

9. How many sq. ft. in the upper surface of 6 paving stones $\frac{3}{4}$ ft. long and $\frac{1}{2}$ ft. wide?

10. What is the number of sq. yd. in the ceiling of a room $18\frac{1}{2}$ ft. long and 16 ft. wide?

If the room is 12 ft. high, what is the number of sq. yd. in the walls of the room?

What is the ratio of the number of sq. yd. to the number of sq. ft.?

11. If a is the number of ft. in the length of a room, b the number in its width, and c the number in its altitude, what equals the number of sq. yd. in the walls of the room? in the ceiling?

12. How much pasteboard is required to make a box $3'' \times 8'' \times 12''$? (No allowance for laps or cover.)

How much is required for the cover, allowing it to turn down 1" all around?

13. How much surface is covered in painting a rectangular prism $3'' \times 7'' \times 11''$?

14. How many sq. in. in the lateral surface of an equilateral triangular prism whose base is 6" on each edge and whose height is 12"?

15. How many sq. in. in the lateral surface of a hexagonal prism whose base is 4" on each edge and whose height is 8"?

16. [1] What is the ratio of the *number* of sq. ft. in a floor 10 ft. $\times$ 12 ft. to the *number* of sq. ft. in a board of flooring 12 ft. long and 4 in. wide?

Then how many such boards are required for the floor?

17. A strip of land containing 2 acres is $\frac{1}{4}$ of a mile long. How wide is it? (See "Elementary Arithmetic," pp. 253–257.)

[1] See number ratios, pp. 251–253, "Elementary Arithmetic."

18. What equals the number of sq. ft. of surface covered in oiling one side of a door 6′ 10″ high by 2′ 4″ wide?

$\frac{7 \cdot 41}{3 \cdot 6}$ = number of sq. ft.

19. How many sq. yd. of plastering in the walls of a room 15 ft. long × 12 ft. wide × 10 ft. high? How many sq. yd. in the ceiling?

Give the dimensions of a rectangle equal to the surface of the four walls. How many sq. ft. in this rectangle? What is the ratio of the number of sq. yd. to the number of sq. ft.?

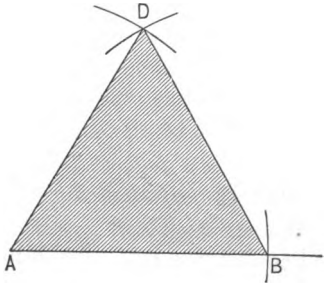
20. What is the ratio of a rectangle $3\frac{1}{2}$″ long and $\frac{1}{4}$″ wide to a sq. in.?

21. What is the ratio of a rectangle $2\frac{1}{4}$″ long and $\frac{1}{2}$″ wide to a sq. in.? What is the ratio of a rectangle $3\frac{1}{7}$″ long and $\frac{1}{4}$″ wide to a sq. in.?

22. At $\$x$ a sq. rd., what equals the cost of a square field whose perimeter is y rd.?

23. If a equals the number of ft. in the length of a room and b the number of sq. ft. in the area, what equals the number of ft. in the width?

24. How many yd. of matting $\frac{7}{8}$ yd. wide are needed to cover a floor 21 ft. square?



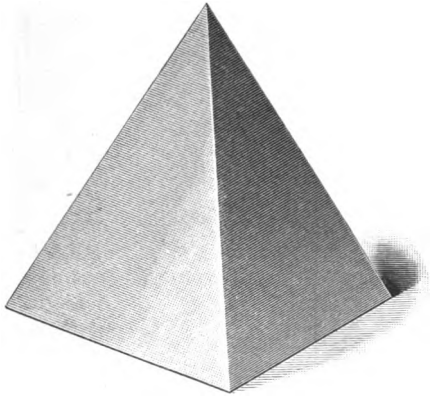
25. At $\$\frac{7}{8}$ a sq. yd., what is the cost of painting a roof $37\frac{1}{2}$ ft. long and 24 ft. wide?

26. At $\$\frac{1}{4}$ a sq. yd., what is the cost of painting both sides of a board fence 110.4 ft. long and 5.25 ft. high?

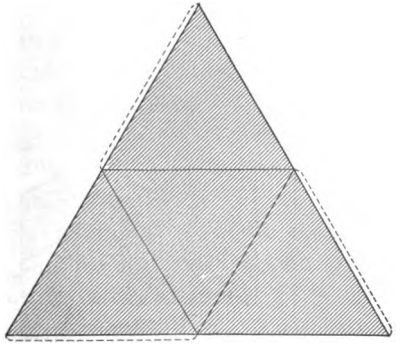
To Draw an Equilateral Triangle. — Let AB be a side of the triangle.

From A and B as centers, and with a radius equal to

AB, describe arcs intersecting at *D*. Join *DA* and *DB*. Practice drawing equilateral triangles. See p. 145.

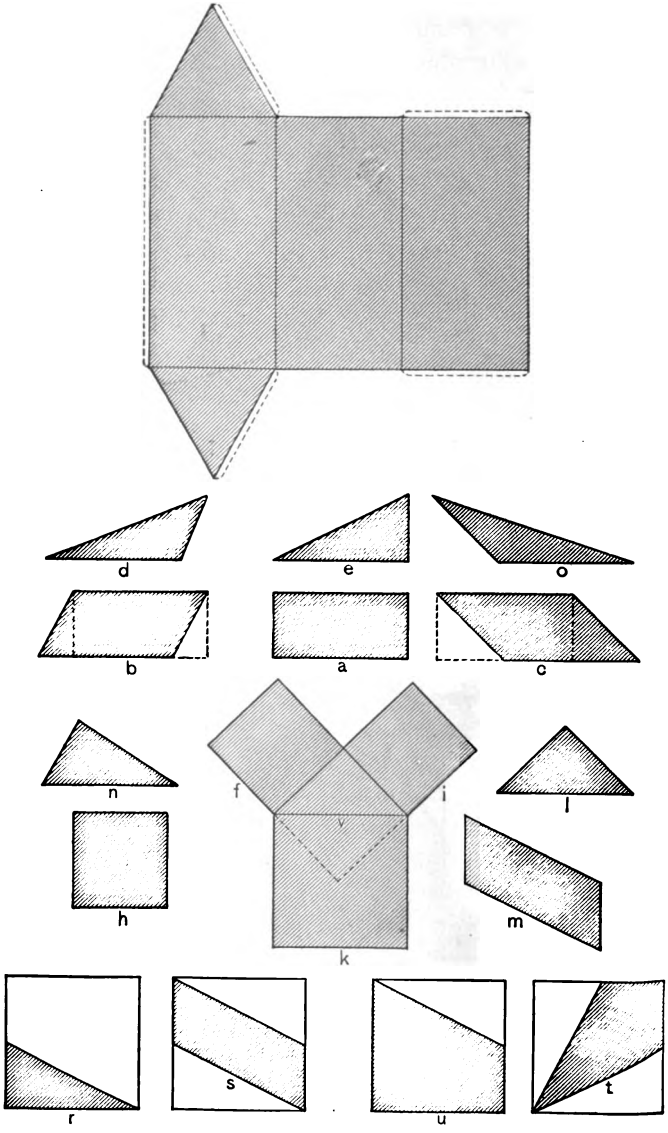


To make a triangular pyramid of which each edge shall be 3 in.

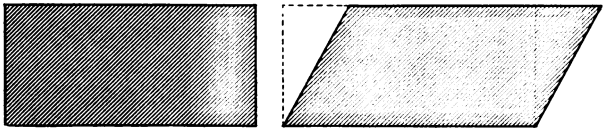


Construct an equilateral triangle whose edge is 6 in.

Separate this equilateral triangle into four equal equilateral triangles.


d
e
o
b
a
c
n
f
i
l
h
k
m
r
s
u
t

1. Find all the ratios you can.[1] See p. 158.
2. Find equal units.
3. Find the ratio 2; the ratio $\frac{1}{2}$.
4. What is the ratio of a to c? of o to c? of a to o?
5. What is the ratio of a to k? of c to k? of o to k?
6. What is the ratio of m to s? of m to r? of m to u? of m to t?
7. What is the ratio of a to s? of l to s? of l to a?
8. What is the ratio of h to the square k? of h to r?
9. What is the ratio of b to k? of h to k? of b to h?
10. h equals $\frac{1}{2}$ of k; b equals $\frac{1}{2}$ of k; therefore h and b are equal. Things that are equal to the same thing are equal to each other.
11. Prove that b and s are equal.



Areas of Rhomboids. — 1. Cut a rectangle having a base of 4 in. and an altitude of 2 in. Cut a rhomboid having a base and altitude equal to those of the rectangle. What is true of the area of the two parallelograms?

2. What is the area of a rectangle whose base is 4 in. and altitude 2 in.? What is the area of a rhomboid whose base is 4 in. and altitude 2 in.?

3. If 17 is the number of ft. in the base of a rectangle and 3 is the number of ft. in its altitude, what equals the number of sq. ft. in its area? If 17 is the number of ft. in the base of a rhomboid and 3 is the number of ft. in its altitude, what equals the number of sq. ft. in its area?

[1] If a portion of several days were given to the study of the relations in the above diagram, it would not result in a loss of time to the pupils.

4. What is the area of a rectangle whose base is 11 in. and altitude 5 in? What is the area of a rhomboid whose base is 11 in. and altitude 5 in.?

5. How many acres in a rectangle 80 rd. long and 14 rd. wide?

$\frac{14 \cdot 80}{160}$ = the number of what?

How many acres in a field in the shape of a rhomboid if its base is 80 rd. and its altitude 14 rd.?

6. Cut rhomboids of different dimensions. Measure base and altitude and find area of each.

7. What are the names of the two kinds of parallelograms?

8. Draw different kinds of parallelograms on the blackboard. Measure base and altitude of each and find the number of units in the area of each.

1. What is the area of a square a in. in length?

2. If a is the number of rd. in the length of a rectangular piece of land and b is the number of rd. in the width, what equals the number of sq. rd. in the area?

3. What is the area of a rhombus whose length is a ft. and whose altitude is b ft.? What is the perimeter of the rhombus?

4. What is the area of a rhomboid a ft. long and b ft. wide?

5. A rectangular piece of land 1 mile long and $\frac{1}{2}$ mile wide contains how many acres?

6. How many rd. of fence will enclose a farm 1 mile square?

7. How much additional fence will divide it into four equal square fields?

8. How many sq. yd. of cloth will it take to cover the tops of all the desks in this room?

9. a is the area of a rectangle and b its altitude. What is its base?

10. a is the area of a rhombus and b its altitude. What is its base?

11. a is the area of a rectangle and b its base. What is its altitude?

12. a is the area of a rhomboid and b its base. What is its altitude?

13. a is the area of a parallelogram and b its altitude. What is its base?

14. A piece of land 40 rd. long and 4 rd. wide contains an acre; how wide is a piece having the same area if it is only $\frac{3}{4}$ as long?

15. An open court contains 40 sq. yd. How many stones 9 in. sq. will be required to pave it?

16. How many sq. in. in the entire surface of a brick 9 in. long, 6 in. wide, and 4 in. thick?

17. What is the length of a parallelogram whose width is 120 rd. and whose area is 60 A.?

18. What is the width of a rectangular field whose length is $16\frac{2}{3}$ rd. and whose area is $12\frac{1}{2}$ A.?

What would the width be if the area were $16\frac{2}{3}$ sq. rd.?

19. The perimeter of a square field equals 160 rd. What is the perimeter of another square field having four times the area of the first?

20. How many steps of 2 ft. 6 in. each will a man take in walking around a field 45 rd. square?

21. How much will it cost to lay a pavement 36 ft. long and 9 ft. 6 in. wide at 40¢ a sq. yd.?

22. How many sq. ft. in a city lot $62\frac{1}{2}$ ft. front and 208 ft. deep?

23. How many tiles 8 in. square will lay a floor 48 ft. by 10 ft.?

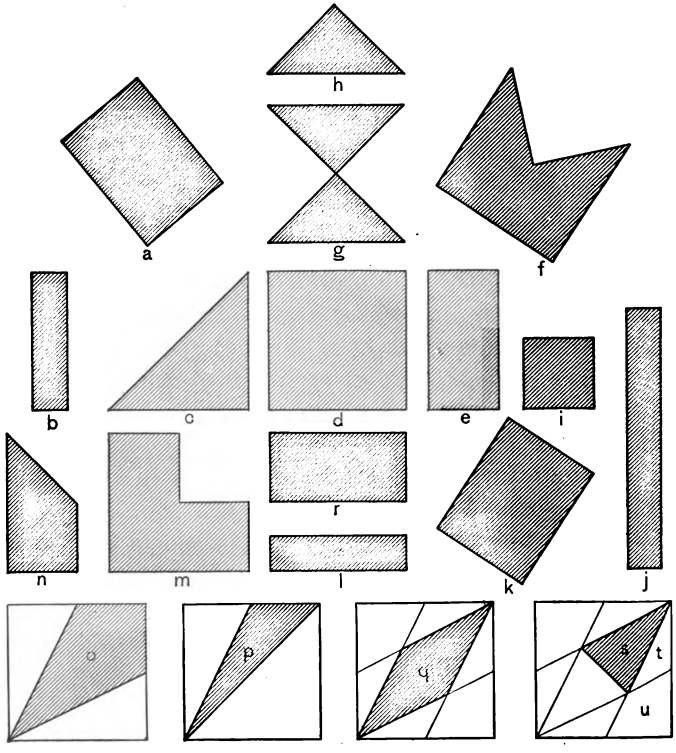
24. At \$2 a rd., how much less will it cost to fence a

piece of land 80 rd. square than a rectangular field twice as long and one-half as wide?

25. What is the area of a field in the form of a parallelogram whose length is 30 rd. and the perpendicular distance between whose sides is 24 rd.?

26. Find the area of the blackboard surface in the room; the area of the ceiling.

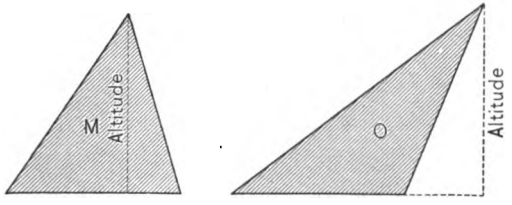
27. A rectangular lawn 18 ft. long and 16 ft. wide has a gravel walk extending around it on the outside. If the walk is 3 ft. wide, how many sq. ft. in it?



1. Find all the ratios you can. See p. 162. Find equals. Find the ratio 2.

2. What is true of t and u? Why?

3. What is the ratio of the sum of t and u to p?



Areas of Triangles. — 1. Measure the base and altitude (height) of each triangle.

2. What is true of the bases of triangles M and O? of their altitudes?

3. What is the base of a triangle? What is the altitude of a triangle?

The base of a triangle is the side measured in finding its area. The vertex of a triangle is the vertex of the angle opposite the base. The altitude of a triangle is the perpendicular distance from its vertex to its base or its base produced.

4. What must be done before the altitude of triangle O can be measured?

5. Draw several triangles. Use as a base a side that must be produced to find the altitude. What is the altitude of each?

6. Measure the base and altitude of each of the parallelograms on p. 164.

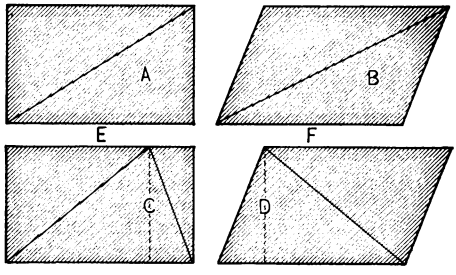
What is true of the bases and altitudes of the parallelograms?

What is true of the areas of the parallelograms?

7. Show lines that are equal. Show parallelograms that are equivalent. Draw two equal parallelograms. Draw

two equivalent parallelograms. What is the distinction between *equal* and *equivalent?* Which of the figures in the diagram are equal and which equivalent?

8. What can be said of the bases of the triangles *A*, *B*,



C, and *D*? of their altitudes? of their areas? Each triangle equals what part of a parallelogram?

9. Cut two equal triangles. Can these two triangles be arranged so that they will make a parallelogram?

10. Can a triangle be made that is not equal to one-half of a parallelogram having an equal base and altitude? Try it.

11. Can you cut or draw a triangle which is not equal to half a parallelogram having an equal base and altitude?

12. The largest triangle that can be cut out of a square equals what part of the square?

13. The largest triangle that can be cut out of a parallelogram equals what part of the parallelogram?

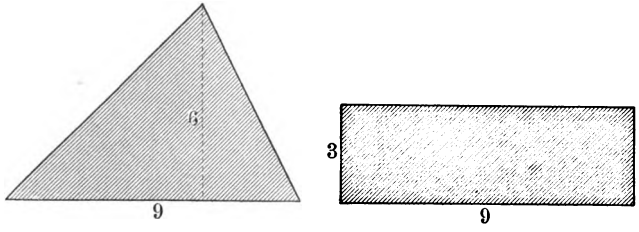
14. What is true of triangles having equal bases and equal altitudes? Why do you think they are equivalent?

15. Draw a triangle. Draw a rectangle having an equal base and an equivalent area.

16. Draw a rectangle. Draw a triangle having an equal base and half the area.

17. What is the altitude of a rectangle 9 in. long that is equivalent to a triangle whose base is 9 in. and altitude 6 in. ?

18. Draw a number of triangles of different sizes and shapes. Draw lines that are equal, respectively, to the



bases of the triangles. On these lines as bases construct rectangles that are equivalent respectively to the triangles.

19. Image rectangles equivalent to triangles whose bases and altitudes are as follows :

Base.	*Altitude.*
12	6
10	7
14	8
$12\frac{1}{2}$	3

20. Find the areas of each of the above triangles.

21. Cut,and find areas of triangles.

22. What is the area of a triangular field whose base is 48 rd. and whose altitude is 28 rd. ?

23. The area of a triangle is 56 sq. in.; its base is 8″. What is its altitude ?

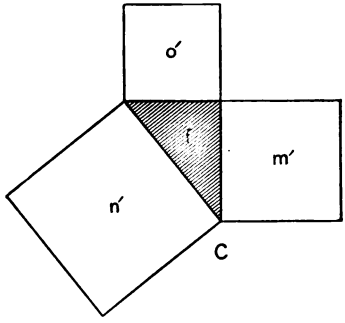
24. A triangular field contains 36 A.; its base is $\frac{1}{4}$ of a mile. What is its altitude ?

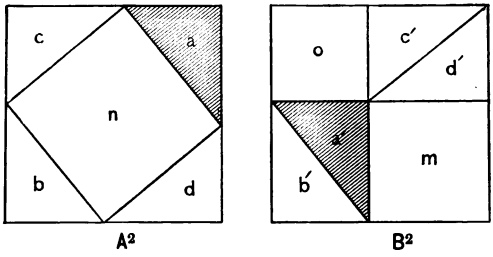
1. Find ratios. See p. 166.
2. Find units that are equal.

3. What is the ratio of A^2 to B^2? of n to the sum of o and m?

4. What is the ratio of n' to the sum of o' and m'?

5. Construct equal squares. Separate into squares and triangles, as in A^2 and B^2. Can you construct a square on the long edge of a right triangle that is not equal to the





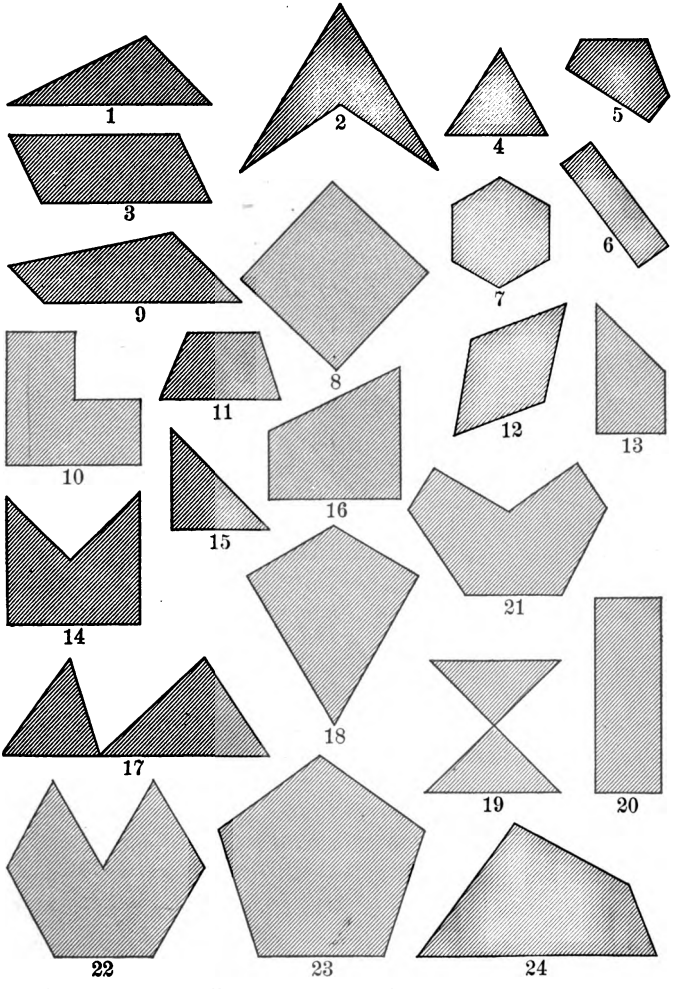
sum of the squares on the other two sides? Is the square of the hypothenuse of a right triangle equal to the sum of the squares on the other two sides?

6. In C, what equals the area of n'? If the area of m' and of n' are given, how can the area of o' be found?

7. If the length of o' and of m' are given, how can the area of n' be found?

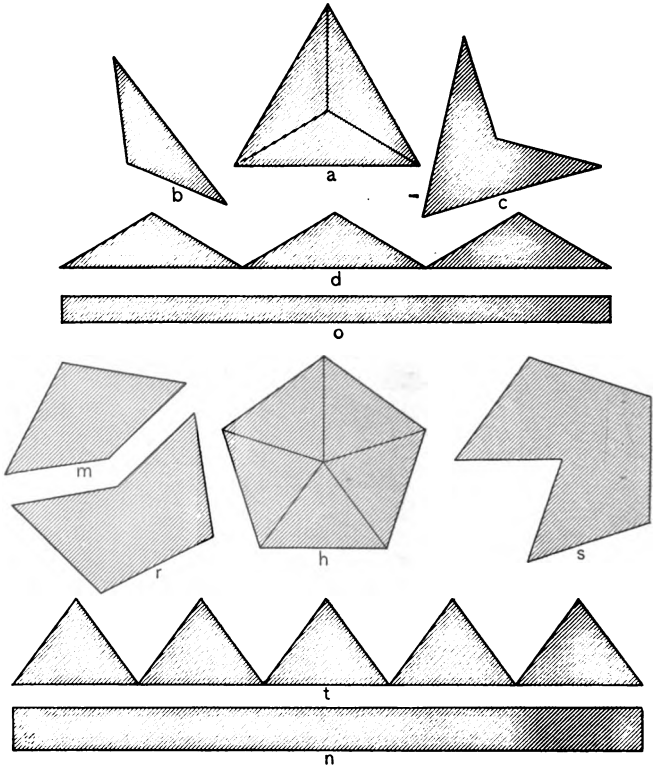
8. If the hypothenuse of f and the side of o' are given, how may the area of m' be found?

Areas of Polygons. — Draw polygons on the blackboard similar to the following:



1. Discover different ways of finding the areas of each.

2. Explain method of finding areas, then measure the polygons and find areas.



1. Find all the ratios you can.

2. What is the ratio of a to d? of d to o? of a to o? of c to o? of b to o?

3. What is the ratio of h to t? of h to n? of m to n? of s to n? of r to n?

4. How may the area of n be found? of t? of h?

5. Draw a line equal to the sum of the bases in diagram t.

On this as a base construct a rectangle equal to the sum of the five triangles.

6. The base of one triangle is 9″, of another 13″; the altitude of each is 8″. How can a rectangle equivalent to the sum of the triangles be drawn?

7. Give a short method of finding the area of a number of triangles having equal altitudes.

Triangles.—1. Draw a triangle a in. long and b in. high; draw a rectangle a in. long equivalent to the triangle.

2. Draw a rectangle equivalent to the area of two triangles whose bases are each a in. and whose altitudes are each b in.

3. If a is the base of a triangle and b is its altitude, what equals its area?

4. If a is the area of a triangle and b its altitude, what equals its base?

5. If a is the area of a triangle and b its base, what equals its altitude?

6. Draw a triangle 2 in. long and $1\frac{1}{2}$ in. high; draw a rectangle 2 in. long equivalent to the triangle.

7. Draw a rectangle equivalent to the area of three triangles whose bases are 2 in., 3 in., and 4 in., respectively, the altitude of each being 4 in.

8. Divide a triangle into three equal parts and state why they are equal.

9. Prove that if equals be taken from equals, equals remain.

10. What is the ratio of a rectangle a in. long and b in. high to the sum of two triangles each a in. long and b in. high?

11. What is the ratio of a triangle whose base is x in. and altitude y in. to a rectangle whose base is x in. and altitude y in.?

12. What is the ratio of a rectangle whose base is 12 in. and altitude 8 in. to a triangle whose base is 12 in. and altitude 16 in.?

13. What may be the dimensions of a rectangle equivalent in area to a triangle whose base is 12 in. and altitude 6 in.?

14. What may be the dimensions of a triangle (base and altitude) equivalent in area to a rectangle whose base is 10 in. and altitude 4 in.?

15. What may be the dimensions of a rectangle equivalent in area to three triangles whose bases are 4 in., 5 in., and 6 in., respectively, the altitude of each being 8 in.?

16. What are the dimensions of a square whose perimeter is 12 in.?

17. What equals the area of a rhombus whose perimeter is a and altitude b?

18. What may be the dimensions of a rectangle whose area is equivalent to that of a triangle having a base of 6 in. and an altitude of 5 in.?

19. What is the perimeter of a square equivalent in area to a triangle whose base is 10 in. and altitude 5 in.?

20. What is the ratio of a square 6 in. long to a triangle twice as long and one-half as high?

21. What is the ratio of a rhomboid whose base is 4 in. and altitude 2 in. to a triangle having an equal base and an equal altitude?

22. What may be the dimensions of a rhombus equivalent in area to a triangle whose base is 12 in. and altitude 6 in.?

23. What is the altitude of a triangle 12 in. long equivalent in area to a parallelogram 12 in. long and 8 in. high?

24. What equals the sum of the areas of two triangles, one a in. by b in., and the other c in. by d in.?

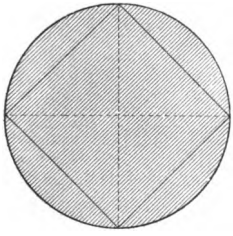
Ans. $\frac{a \cdot b}{2} + \frac{c \cdot d}{2}$ = area of two triangles.

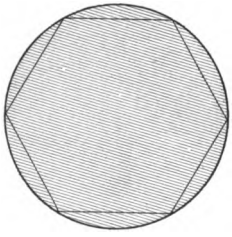
25. What equals the difference in area of two triangles, the smaller one being m ft. by n ft., and the larger x ft. by y ft.?

$$Ans. \quad \frac{x \cdot y}{2} - \frac{m \cdot n}{2} = \text{difference.}$$

Constructions. — 1. To inscribe a square in a circle. See note, p. 145.

Draw two diameters perpendicular to each other. Join their extremities by chords.





Practice inscribing squares.

2. To inscribe a regular hexagon in a circle, apply the radius as a chord six times to the circumference.

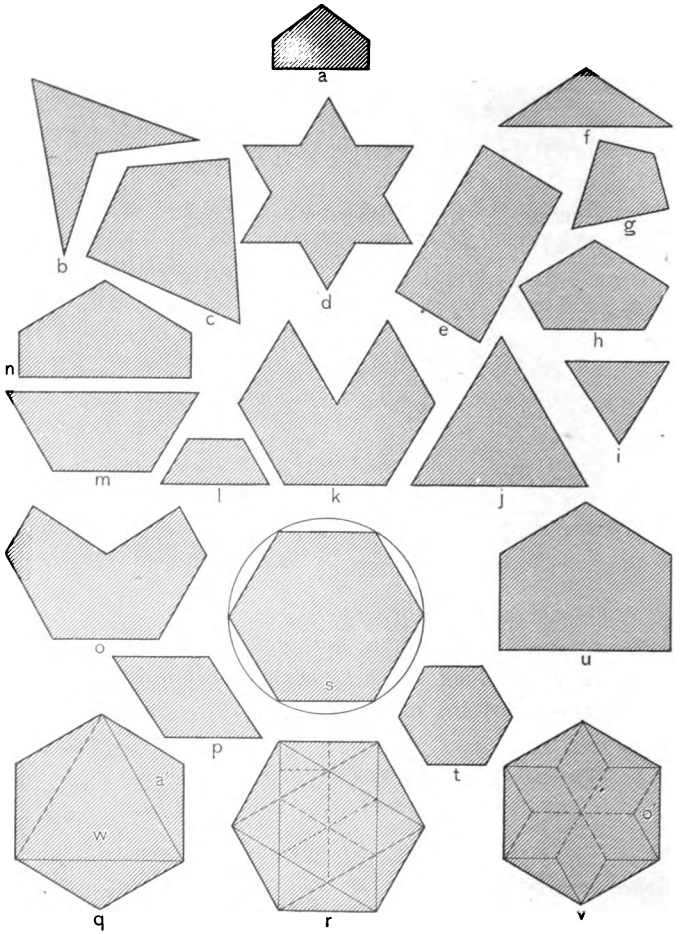
Practice inscribing hexagons.

1. Find all the ratios you can. See p. 172.
2. a' equals what part of q?

"In mathematics I had many truths put before my eyes, and concluded many others from them by analogy." — DESCARTES.

"The most obtrusive form of matter is the solid, and for this reason it is that form which is first cognized by the infant intellect of mankind, and thus serves as the basis for the subsequent recognition of other forms." — J. B. STALLO.

"Among the Greeks, as in the school of Pestalozzi, . . . mathematics were drawn back to the primary elements of education." — HAMILTON.

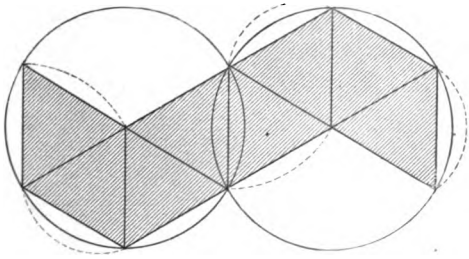


3. What is the ratio of o' to a'?

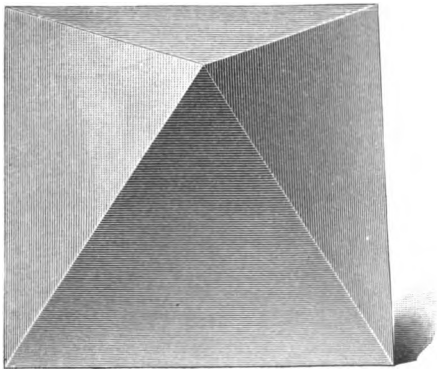
4. What is the ratio of each form to the hexagon r?

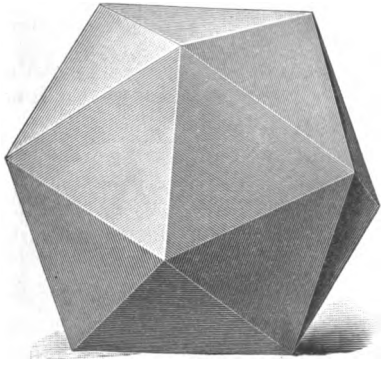
5. Find equal units.

6. l equals what part of r? i equals what part? What is the ratio of l to i?

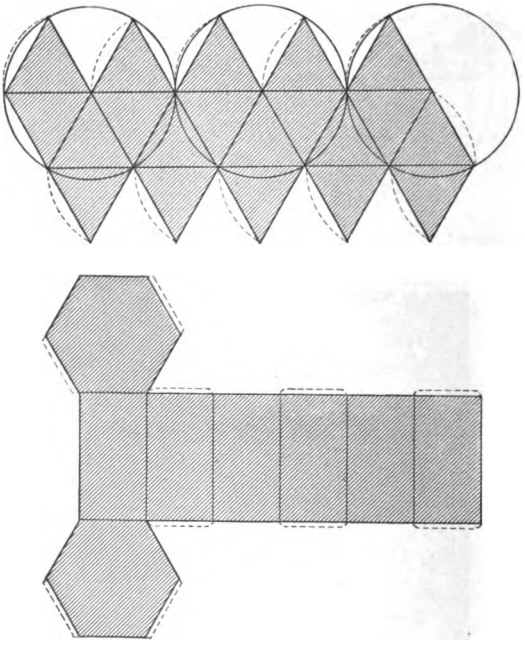


Practice constructing octohedrons.





Practice constructing icosahedrons.



"Imagination expands, diminishes, molds, and refines, as the case may be, materials derived from the world of fact and observation." — JOHN TYNDALL.

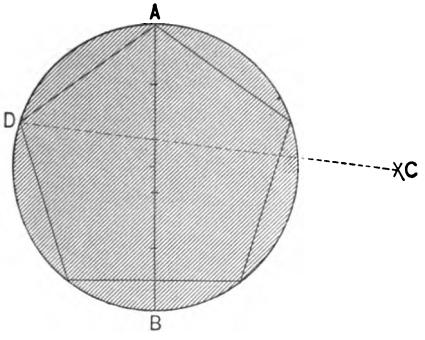
"It is quite a mistake to consider what Tyndall calls imagination as mere fancy. It is exactly the opposite that is meant — full, sensuous contemplation." — HELMHOLTZ.

"The kingdom of science, then, cometh not by observation and experiment alone, but is completed by fixing the roots of observation and experiment in a region inaccessible to both, and in dealing with which we are forced to fall back upon the picturing power of the mind." — JOHN TYNDALL.

"The greater the power of abstraction and generalization which is observed, the greater is the power of representation which is implied." — JOHN FISKE.

Construct a hexagonal prism.

General method of inscribing any regular polygon in a circle shown by inscribing a regular pentagon.

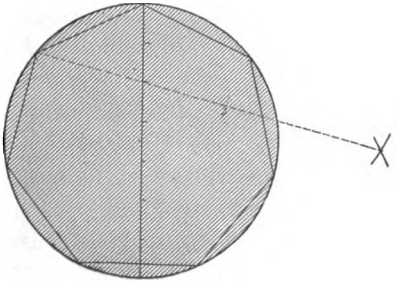


Use the dividers and mark off five equal divisions on a line.

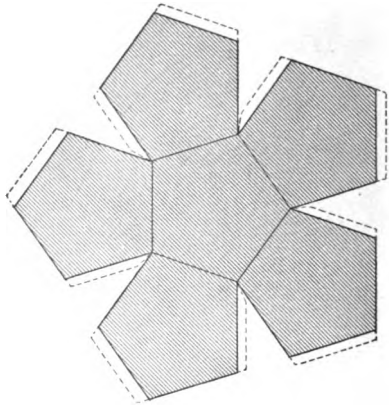
Describe a circumference, using half the sum of the five divisions as a radius.

From A and B as centers, and with AB as a radius, describe arcs intersecting at C. Draw the line CD through the second division of AB. The line DA is the side of the pentagon.

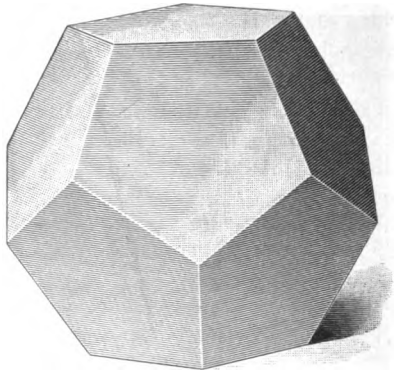
Any regular polygon can be inscribed by the method given above by making the number of divisions of the diameter equal to the number of the sides of the polygon and drawing CD through the second division of the diameter.

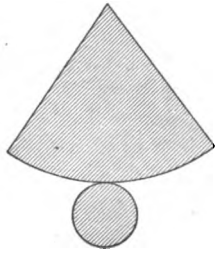


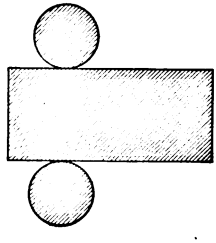
This diagram represents one-half of the surface of a dodecahedron.



Construct dodecahedrons. Construct cones and cylinders.





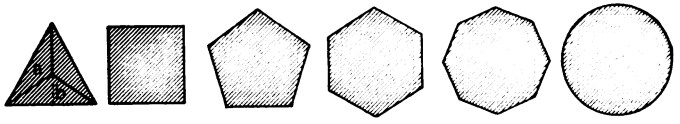


1. Practice drawing the following (see method, p. 175): A regular hexagon; a regular heptagon; a regular octagon, etc.

2. In and out of school find regular polygons. Observe the drawing models.

3. Draw pentagon and five-pointed star.

4. Pupils give directions for inscribing figures and other pupils draw figures. Inscribe the figures *mentally* according to oral directions, and tell what polygon is inscribed.



Regular Polygons. — 1. In what are the above regular polygons alike?

2. What five things in each of the regular polygons are respectively equal?

3. *A* is a greater radius of a regular polygon. Show greater radii in the other polygons. What is a greater radius?

4. Into what do the greater radii of a regular polygon separate the polygon?

5. *B* is a less radius. Find less radii of other polygons. What is a less radius?

6. In which regular polygon is there the greatest difference in the lengths of the greater and less radii?

7. As the number of sides of the polygons increase, what change takes place in the relative lengths of the greater and less radii?

8. Which line of the regular polygon is the altitude of the triangles into which it may be separated?

9. Can you construct a polygon with equal greater and equal less radii and not make it regular?

10. If the greater and less radii of any polygon are respectively equal, what kind of a polygon is it?

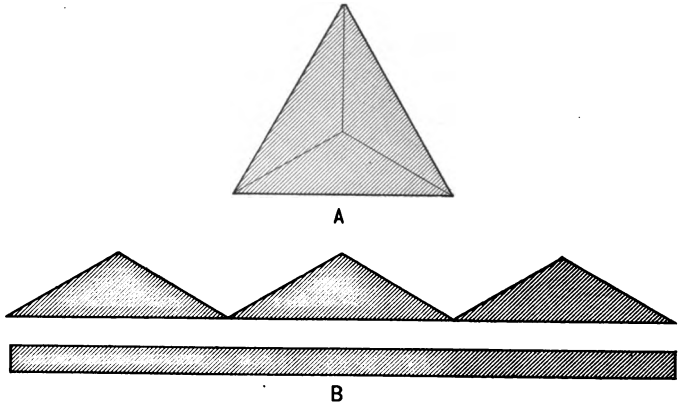
11. What is a regular polygon?

A regular polygon is a polygon in which the greater and less radii are respectively equal.

12. In which regular polygon are all the radii equal?

A circle is a regular polygon whose radii are equal.

13. How may the area of any regular polygon be found?



Regular Polygons and Equivalent Rectangles. — 1. Triangle A is separated into how many equal triangles?

2. Show the circumference of A. Which line of rectangle B equals the circumference of A? The altitude of B equals one-half of what line in A? What is true of A and B?

3. Draw a regular pentagon. Draw a line equal to the circumference of the pentagon. On the line, construct a rectangle equivalent to the pentagon.

4. Draw a hexagon. Draw a line equal to the circumference of the hexagon. On the line, construct a rectangle equivalent to the hexagon.

5. Draw a regular heptagon. Draw a less radius of the heptagon, but do not separate the heptagon into triangles. Draw a line equal to the circumference of the heptagon. Construct on the line a rectangle equivalent to the heptagon.

6. If the number of units in the circumference of a regular polygon is given, what other number is required in order to find the area of an equivalent rectangle?

The number of units in the area of the polygon equals the product of what numbers? Why?

7. Is a circle a regular polygon? When the number of units in the circumference of the circle is known, what other number is needed to find its area?

The product of what numbers equals the number of units in the area of the circle?

8. If a is the perimeter of an equilateral triangle and b its less radius, what equals its area?

9. If a is the perimeter of a square and b its less radius, what equals its area?

10. If a is the perimeter of a regular hexagon and b its less radius, what equals its area?

11. If a is the perimeter of a regular octagon and b its less radius, what equals its area?

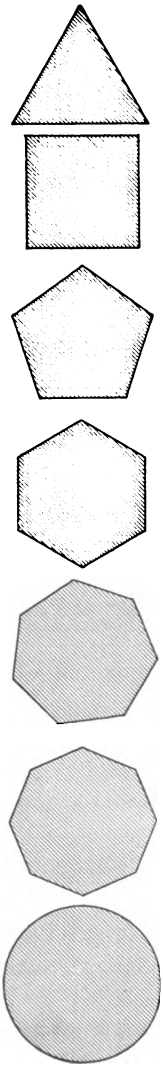
12. If a is the circumference of a circle and b its radius, what equals its area?

13. If the length of one side of an equilateral triangle is a and its less radius b, what equals its area?

14. If the length of one side of a square is a, what equals its area?

15. What is the ratio of the perimeter of a square to its less radius?

Areas of Regular Polygons. — 1. The circumference of a square equals how many times its less radius?



2. If 5 is the less radius of a square, what is its circumference?

The less radius equals what part of the circumference of a square?

3. If 56 is the circumference of a square, what is its less radius? if 19? if x?

4. What is the ratio of the less radius of a square to its circumference?

5. What is the radius of the largest circle that can be inscribed in a square 8″ in diameter?

6. What is the less radius of a square 5 ft. long? of a square 1 mile in circumference? 1 yd. in circumference? $\frac{2}{3}$ in. in circumference? .35 ft. in circumference?

7. What is the less radius of a square whose circumference is 100?

8. What is the less radius of a square whose area is 16? 25? 49?

9. Draw a rectangle which is equivalent to a square and which has a base equal to the circumference of the square. What is its altitude? What is the area of this rectangle?

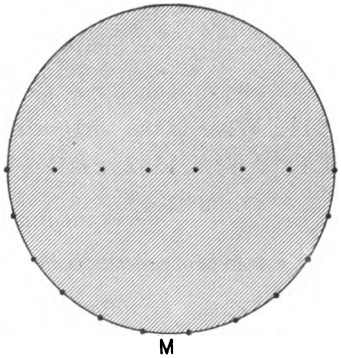
10. In the manner of the last problem find the area of a 6″ square; a 12″ square; a 3 ft. square; a 1″ square; a 9″ square; a square $5\frac{2}{3}$″ long.

11. The perpendicular distance from the center of a square floor to one of its sides is 12 ft. What is the area of the floor? What is the length of the perimeter of the floor? If the room is 13 ft. high, what is the area of the walls of the room?

1. Cut a circle having a 7″ diameter. Measure the circumference carefully.

Suggestion. — Bend a strip of paper around half of the circumference, then measure the paper. Twice the length of the paper equals the circumference.

2. What is the length of the circumference of a 7″ circle? The circumference of a circle is how many times as long as $\frac{1}{7}$ of its diameter? The diameter equals how many twenty-seconds of its circumference?



3. How many equal divisions in the diameter of the circle *m*?

How many divisions equal to $\frac{1}{7}$ of the diameter are found in $\frac{1}{2}$ of the circumference? in the circumference?

4. What is the ratio of the circumference of a circle to its diameter? What is the ratio of the diameter to the circumference?

5. If the diameter of a circle is known, what can be found? How?

6. If the circumference of a circle is given, what can be found? How?

7. Draw a straight line equal to the circumference of a circle $3\frac{1}{2}$″ in diameter.

8. Draw a straight line equal to the circumference of the circle $1\frac{3}{4}$″ in diameter.

9. The circumference of a circle equals how many diameters?

What is the ratio of the circumference of a circle to its diameter?

Note.— $3\frac{1}{7}$ is an approximate value of the ratio of the circumference of a circle to its diameter. Its absolute value cannot be expressed. For exact computations this value has been carried out many decimal places. $3\frac{1}{7}$ is accurate enough for most purposes. For more precise estimates the decimal 3.1416 may be used. This ratio may be denoted by the Greek letter π (pi); thus, the circumference of the circle may be indicated by $\pi\, d$ or $2\pi\, r$ (d diameter, and r radius).

10. What is the circumference of a circle whose diameter is 14″? 21″? 9″? 42″? $6\frac{1}{2}$″? $11\frac{1}{5}$″? $\frac{5}{8}$″? 84″? 1.38″?

Ex.:— $\frac{22 \cdot 14}{7}$ = number of in. in the circumference of a circle whose diameter is 14″.

11. What is the circumference of a circle whose radius is 14″? 6″? 13″? 21″? $\frac{7}{3}$″? $\frac{14}{2}$″? $\frac{7}{5}$″? $1\frac{1}{7}$″?

Ex.: — $\frac{22 \cdot 2 \cdot 14}{7}$ = the number of in. in the circumference of a circle whose radius is 14″.

Draw circles of different sizes on the blackboard. Measure diameters. Pupils in seats find circumferences.

12. If the hour-hand of a clock is 5″ long, how far does its extremity move in 12 hr.? in 8 hr.? in $1\frac{1}{2}$ hr.?

13. A horse is tied to a stake by a strap 8 rd. long. What is the circumference of the circle in which he has grazed?

14. The diameter of a circle is 18 rd. How far will John have to travel to pass around the circle if he starts at the center?

15. What is the circumference of a cylinder if its diameter is 7″? 15″? $\frac{1}{8}$″?

16. What is the diameter of a circle whose circumference is 3″? $3\frac{1}{2}$″? 7″? 8″? 1 ft.? 1″? 98″? 49″? 22 ft.? 3 ft.?

17. The diameter of a circle is 7″; what is the length of an arc of 75°? of 170°?

18. What is the length of an arc of 75° in a circle whose diameter is 16 ft.?

19. What is the diameter of that circle the length of whose arc of 80° is 44 in.?

20. How many degrees in the arc whose length is 66 in. if the diameter of the circle is 60 in.?

21. What is the diameter of a tree whose circumference is 20″? What is the diameter of a log whose circumference is 6 ft.?

22. Draw a circle whose circumference is $5\frac{1}{2}$ ft.

Draw a straight line equal to the circumference, then a line equal to the diameter.

How long is the radius? Describe the circle.

23. Draw a line equal to the circumference of a 6″ circle.

24. A line $2\frac{3}{4}$ ft. long equals the circumference of a circle whose diameter is how many inches?

25. Draw a circle having a circumference of 11 in.

26. If 7 in. is the greatest distance through a ball, what is the greatest distance around the ball?

27. If 3 in. is the diameter of a croquet ball, what is its circumference?

28. If 25 is the number of units in the area of a square, what is the square root, or the number of units in its length?

29. What is the square root of 17? (*Ans.* 4 +.) of .17? (*Ans.* .4 +.)

30. What is the square root of $\frac{1}{4}$? of $\frac{1}{9}$? of $\frac{4}{9}$? of $\frac{9}{16}$?

31. What is the square root of 16? of 36? of 64? of 49? of 81? of 84? of .84?

32. $\sqrt{144} = 12$, $\sqrt{169} = ?$ $\sqrt{72} = ?$ $\sqrt{13} = ?$ $\sqrt{95} = ?$ $\sqrt{147} = ?$ $\sqrt{400} = ?$

1. Draw the square of the d of the circle whose d is $1''$; $2''$; $2\frac{1}{2}''$.

2. A square $1''$ long equals what part of a square $2''$ long? of a square $3''$ long? A square $2''$ long equals what part of a square $3''$ long?

3. A square $\frac{1}{2}''$ long equals what part of a square $1''$ long?

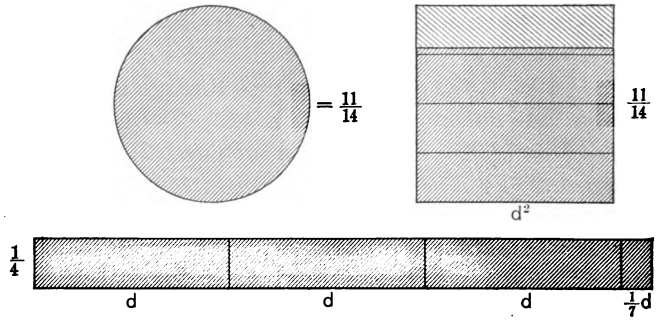
4. A square $\frac{3}{5}''$ long equals what part of a square $1''$ long? $2''$ long?

5. The square of the radius of a circle equals what part of the square of the diameter?

6. A rectangle $3\frac{1}{7}''$ long by $\frac{1}{4}''$ wide equals what part of a square inch?

7. If d is the diameter of a circle, what is its circumference?

8. If b is the circumference of a circle, what is its diameter?



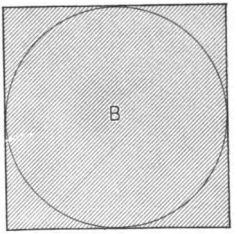
9. Draw a circle. Draw a straight line equal to the circumference of the circle. On the straight line construct a rectangle equivalent to the circle.

10. Draw the square of the diameter of the circle.

11. The rectangle equivalent to the area of the circle equals what part of the square of its diameter? What is

the ratio of the area of the circle to the square of its diameter? Of any circle to the square of its diameter? What is the ratio of d^2 to the area of the circle?

12. What is the area of a circle whose diameter is 7″? What is the square of the diameter? The circle equals what part of the square of its diameter? $38\frac{1}{2}$ sq. in. equals what part of 49 sq. in.?



13. The circle B is what part of the square of its diameter? The sum of the parts of the square outside of the circle equals what part of the square? what part of the circle?

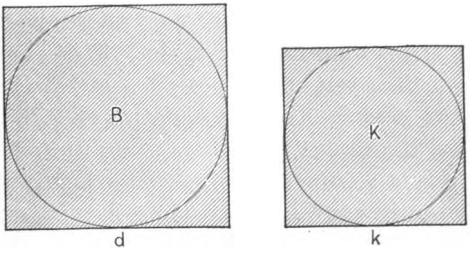
14. If 77 is the number of units in the area of the circle B, what equals the number in the area of the square?

15. If 21 is the number of units in the diameter of a circle, what equals the number of units in its area?

$\frac{11 \cdot 21 \cdot 21}{14}$ = the number of units in its area.

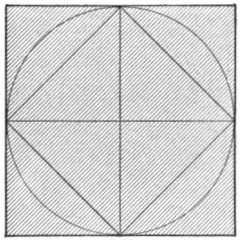
$21 \cdot 21$ = the square of what? $\frac{11}{14}$ of 21^2 = what?

16. What equals the number of units in the areas of



circles having the following diameters: 8″, 14″, 24″, 85″, .7″, 2.1″, $3\frac{1}{2}$″, $\frac{7}{11}$″, 1.38″?

17. Find all the ratios you can. What is the ratio of d to k? The circle inscribed in d equals what part of d? The circle inscribed in k equals what part of k? What, then, is the ratio of the larger circle to the smaller? Why?



18. What is the ratio of circles having the following diameters: 1″ and 2″? 2″ and 4″? 2″ and 3′? 2′ and 5′? $\frac{1}{2}$″ and $\frac{2}{5}$″? $\frac{3}{10}$′ and $\frac{3}{5}$′?

19. What is the relation of the circle to the inscribed square and to the circumscribed square? Of the circumscribed square to the circle? to the inscribed square? Of the inscribed square to the circle? to the circumscribed square?

20. Draw circles on the blackboard. Measure their diameters and compute areas.

21. Make many measures of the diameters of cylinders, cups, barrels, pails, bases of cones, plane surfaces of hemispheres, dials of clocks and watches, etc. Compute the areas of the circles whose diameters are measured.

22. A sector is a part of a circle bounded by two radii and an arc. Show me a sector.

A boy receives a sector of a pie having an arc of 100 degrees. What part of the pie does he receive?

23. If the minute hand of a clock is $5\frac{1}{2}$″ long, what is the area of the circle that it describes in 1 hr.?

24. Draw a circle having a diameter of 2.76″. What is the area of the circle?

25. From a rectangle 2″ by 3″, how many circles 1″ in diameter can be cut? After cutting the circles, what part of the rectangle remains?

26. If a is the diameter of a circle, what equals its area?

27. The square of the diameter of a circle equals what part of the square of its circumference?

28. The diameter of the base of a cylinder is $3\frac{3}{4}''$; its altitude is $7''$; what is its entire surface?

29. If a is the radius of a circle, what is its area?

30. If a is the area of a circle, what equals the diameter of the circle?

31. If a is the area of a circle, what equals the radius of the circle?

32. If a is the square of the radius of a circle, what equals the area of the circle?

33. If a is the square of the diameter of a circle, what equals the area of the circle?

34. If a is the perimeter of the square of the diameter of a circle, what equals the area of the circle?

35. To what rectangle is a circle equivalent?

36. To what rectangle is any regular polygon equivalent?

37. What are the dimensions of a rectangle equal in area to a circle whose diameter is 1 in.?

38. What may be the dimensions of a rectangle equal in area to a circle whose radius is $\frac{1}{2}$ in.?

39. What may be the dimensions of a rectangle equal in area to a circle whose diameter is 2 in.?

40. If a is the square of the radius of a circle, what is the square of its diameter?

41. If a is the square of the diameter of a circle, what equals the square of its radius?

42. What is the area of the largest circle that can be inscribed in a 2-in. square?

43. What is the area of the largest circle that can be inscribed in a room x ft. long and 7 ft. wide?

44. What is the circumference of a circle that can be

traced by a pair of compasses whose points are x inches apart?

45. What equals the area of the circle in question 44?

46. What is the area of the largest circle that can be inscribed in a 3-in. square?

47. What is the ratio of a circle inscribed in a 1-in. square to a circle inscribed in a 2-inch square?

48. What is the ratio of a circle inscribed in a 2-in. square to a circle inscribed in a 4-inch square?

49. What is the area of a circle whose circumference is 88″?

50. If this room is 30 ft. long and 25 ft. wide, what is the area of the largest circle that can be described in the room?

51. What is the area of a railroad turn-table 35 ft. in diameter?

52. The square of the radius of a circle equals what part of the square of the diameter?

53. A log is 15 ft. long and the diameter of each end is $4\frac{2}{5}$ ft. What is its entire surface?

54. What is the relative size of two circles, one of which has a diameter of $\frac{3}{5}$″ and the other a diameter of $\frac{4}{5}$″?

55. If x is the radius of a circle, find area.

56. If x is the diameter, find radius. Find circumference. Find area.

57. If x is the circumference of a circle, find diameter, radius, area.

58. If a is the diameter of the base of the cylinder and b is its altitude, find its lateral surface; its entire surface.

59. If a is the diameter of the base of a cone and b the slant height, find lateral surface; entire surface.

60. What is the entire surface of a cylinder whose base is 13″ in diameter and whose altitude is 17″? Find the lateral surface.

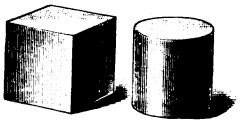
61. What is the entire surface of a cone whose base is 13″ in diameter and whose slant height is 17″?

62. What is the entire surface of a cylinder if the diameter of the base is 1 ft. and the altitude 1 ft.?

63. A circle whose diameter is 1″ equals what part of a circle whose diameter is $1\frac{1}{2}$″?

64. Give two methods of finding area of a circle.

Volumes of Cylinders. — 1. The largest cylinder that can be cut or turned out of a cubic inch equals what part of the cubic inch?



2. The largest cylinder that can be turned out of any square prism equals what part of it?

3. What equals the number of cubic inches in the largest cylinder that can be turned out of a cubic foot?

4. Give a rule for finding the volume of a cylinder.

5. Review from the first of mensuration.

6. What is the volume of a cylinder whose altitude is 5″ and the diameter of whose base is 2.76″?

7. What is the volume of a square prism 7″ long and 14″ high?

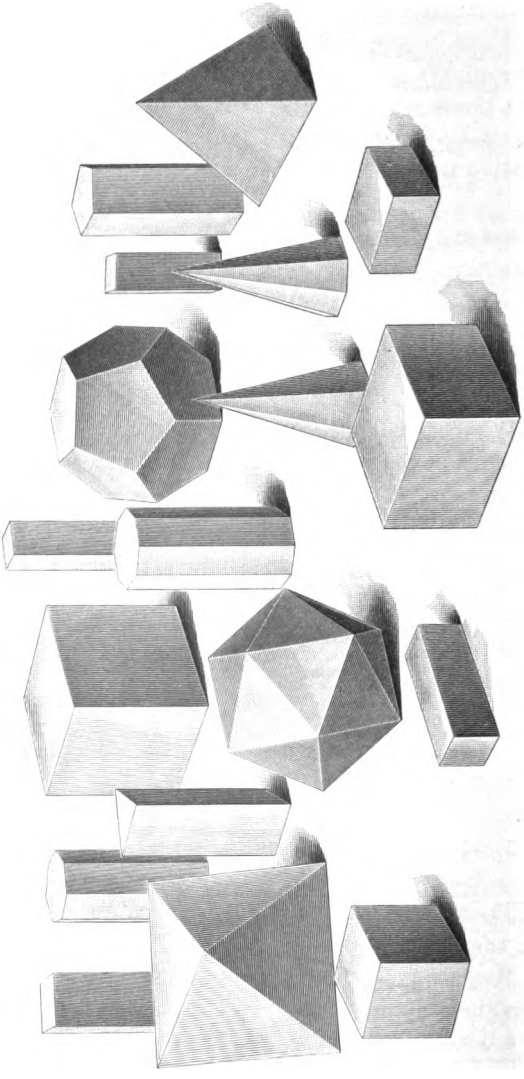
8. What is the volume of a cylinder whose altitude is 14″ and the diameter of whose base is 7″?

9. If b is the altitude of a cylinder and a is the diameter of its base, what equals its volume?

10. If 51″ is the diameter of a cylinder and its altitude is 3.7″, what is its volume?

11. The volume of the cylinder in problem 8 equals what part of the prism in problem 7?

12. How many 2″ cubes can be put into a box $5'' \times 7'' \times 9''$? What is the volume of the largest cylinder that can be put into the box?

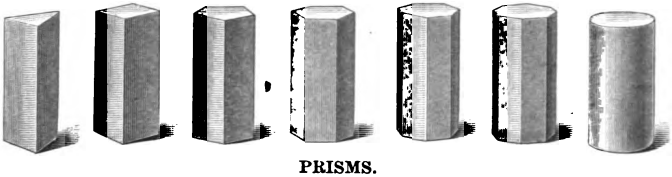


POLYHEDRONS.

Polyhedrons. — What is a geometrical solid? In what are these solids alike?

What is a polygon? By what are polyhedrons bounded? What is a polyhedron?

A polyhedron is a solid bounded by polygons.



PRISMS.

Prisms.—1. In what are these polyhedrons alike? different?

2. What is true of the upper and lower bases of these polyhedrons?

3. Of what is the convex surface of each composed?

4. Show the equal and parallel polygons. Show the parallelograms.

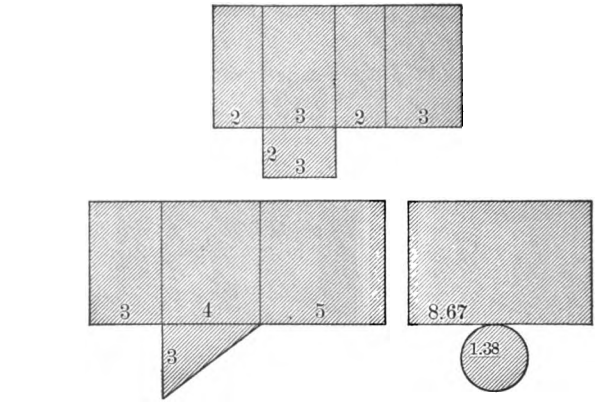
5. These polyhedrons are called prisms. What is a prism? A prism is a polyhedron whose bases are equal and parallel and whose convex surface is composed of parallelograms.

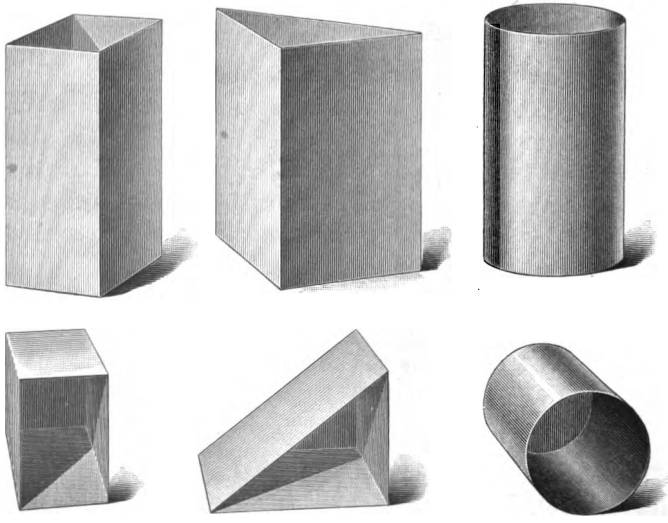
6. What is the least number of lateral surfaces a prism can have? The greatest number?

7. What is the name of the prism having the least number of sides? The greatest number?

8. Is a cube a prism? Why? Is a cylinder? Why?

Volumes of Prisms. — Make of "tough check," or of some other tough card board, a quadrangular, a triangular, and a cylindrical prism. Make their bases equivalent and altitudes equal. The diagrams (see page 192) will aid in making the prisms. Do the work carefully. Compare their volumes by measuring sand, salt, or sugar.





PRISMS.

1. Find the areas of the bases of the prisms.

2. What is true of the bases of the prisms? of their altitudes? of their volumes?

If the *quadrangular* prism were 1″ high, how many cubic inches would it contain?

If the *triangular* prism were 1″ high, how many cubic inches would it contain?

If the *cylinder* were 1″ high, how many cubic inches would it contain?

Each prism is 5″ high. How many cubic inches does each contain?

What is the number of sq. in. in the base of each prism? What is the number of inches in the altitude of each? Then the product of what numbers equals the number of units in the volume of each?

3. What is a prism? When are prisms equivalent?

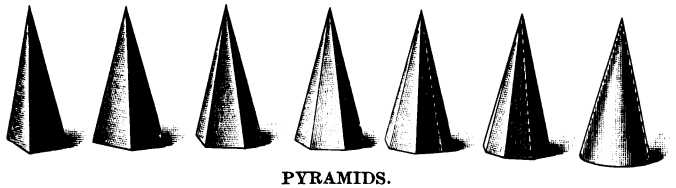
4. What shows the relation of prisms having equivalent bases? having equal altitudes?

5. If a and b are two prisms having equal altitudes and the base of a equals $\frac{2}{7}$ of the base of b, what is the relation of b to a?

Suggestion. — Draw the diagrams on pages 192 and 194, take them to a tinsmith, and have the solids made. The making of the three prisms and of the two pyramids will cost less than 75¢. The comparison of the volume by measuring water will interest the class and explain much of the work that follows.

Pyramids. — 1. In what are these polyhedrons alike?

2. What general name do we give to the base of each?



PYRAMIDS.

3. Of what is the lateral surface of each composed?

4. These polyhedrons are pyramids.

5. The base of which pyramid has the least number of sides? The greatest number?

6. If the base of a cone is a polygon, of what is the lateral surface of the cone composed?

7. What is a pyramid?

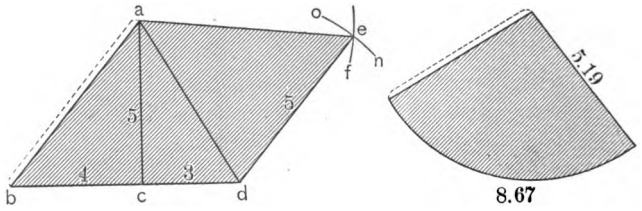
A pyramid is a polyhedron having a polygon for its base and triangles for its lateral surface.

8. What is the altitude or height of a pyramid?

Out of "tough check" make a triangular pyramid and a conical pyramid having equivalent bases and equal altitudes. Compare the volumes of the two pyramids.

The diagrams below show how the tough check may be cut and folded into the required pyramids.

To make a pyramid and a cone having equivalent bases and equal altitudes.



Draw a line *bd*.

Draw *ac* perpendicular to *bd* at *c*.

Draw *ab* and *ad*.

With *a* as a center and *ab* as a radius draw the arc *ef*.

With *d* as a center and a 5″ radius draw the arc *on* intersecting *ef*.

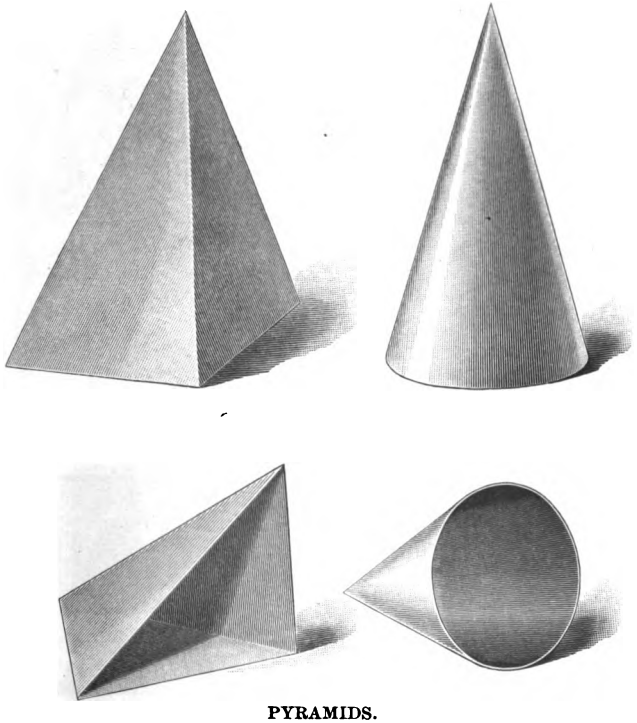
Join *a* and *e* and *de*.

Cut and fold to shape.

What is the altitude of the cone? of the triangular pyramid? What is the base of the cone?

Suggestion. — It may be well to make the slant height of the cone 5.2″ and the circumference of the base 8.7″ instead of the more accurate measures given above.

1. What is true of the bases of the triangular and conical pyramids? of their altitudes? of their volumes?

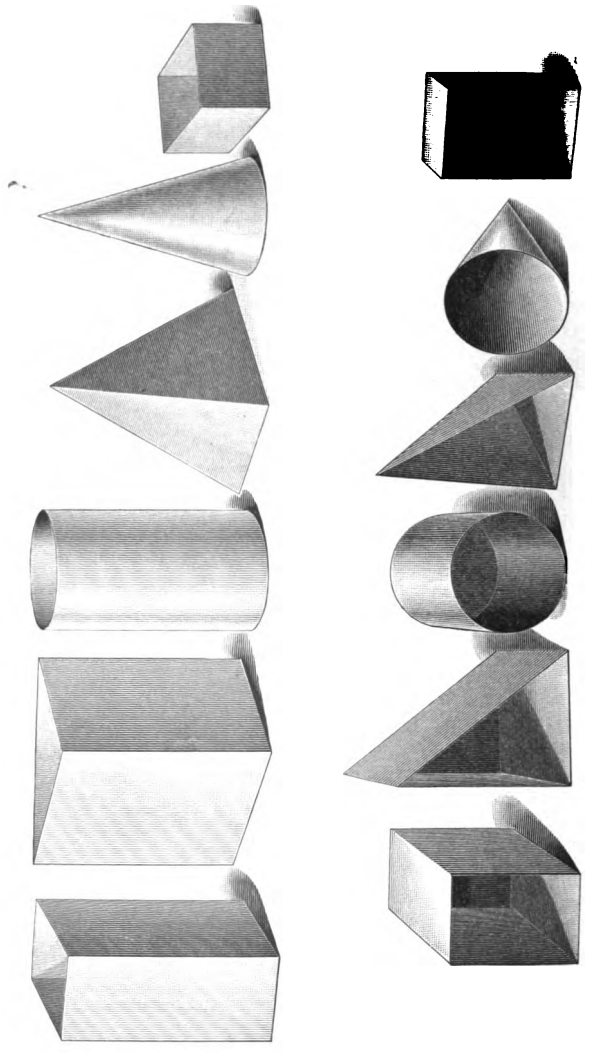


PYRAMIDS.

2. Compare the volumes of the pyramids and prisms by measuring water.

3. What is the ratio of one of the prisms to one of the pyramids?

4. How is the volume of a prism found? How, then, may the volume of the pyramid be found?

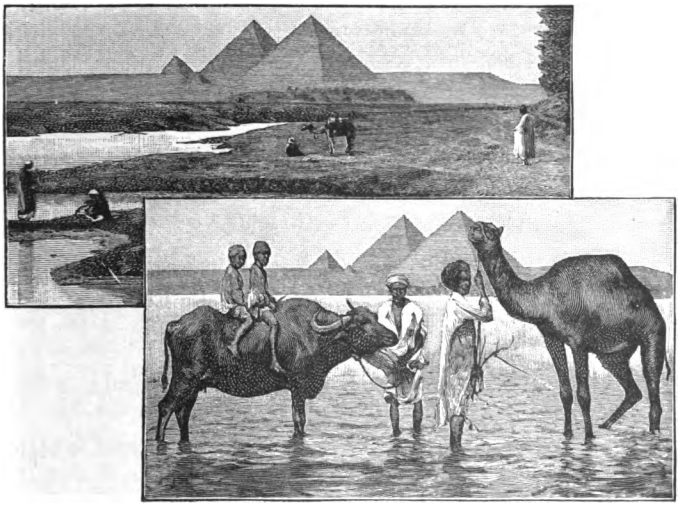


PRISMS AND PYRAMIDS.

5. What is the volume of a pyramid whose base is 6 sq. in. and altitude 5″?

6. The base of a pyramid is 75 sq. in. and its altitude 27″. What is its volume?

7. If the base of a cone is 18 sq. in. and its altitude 9″, what is its volume? What is the ratio of the volume of a prism whose base is 18 sq. in. and altitude 3″ to the cone?



8. If the diameter of the base of a cone is 7″ and its altitude is 8″, what is its volume?

$\frac{8 \cdot 11 \cdot 7 \cdot 7}{3 \cdot 14}$ = the number of units in the volume of the cone.

$\frac{11 \cdot 7 \cdot 7}{14}$ = what? Then $\frac{8 \cdot 11 \cdot 7 \cdot 7}{14}$ equals what? Then

$\frac{8 \cdot 11 \cdot 7 \cdot 7}{3 \cdot \quad 14}$ = what?

9. Find the volumes of the following cones:

Diameter of Base.	*Altitude.*
21″	12″
63″	18″
15″	17″
$3\frac{1}{2}$″	5″
.9″	1.1″
.18″	.84″

10. Make and solve five problems similar to those given above. Measure the diameters of the bases and the altitudes of cones and find volume.

11. Draw a quadrangular prism equal to the sum of the following pyramids: The altitude of each is 6″; the bases are respectively 7 sq. in., 5 sq. in., and 3 sq. in.

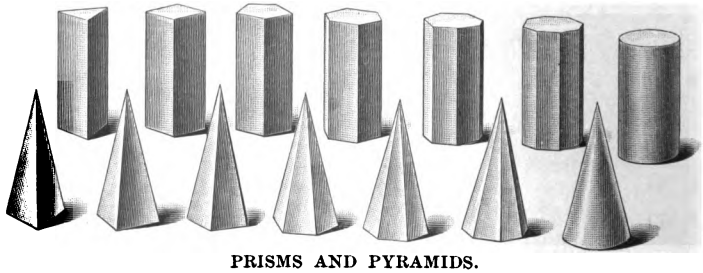
12. Draw a quadrangular prism equal to a pyramid whose base is 6 sq. in. and whose altitude is $1\frac{1}{2}$″.

13. Give a method of finding the volume of three pyramids of equal altitude. Would this do? Multiply the number of units in the sum of their bases by $\frac{1}{3}$ of the number of units in their altitude.

14. Given 100 pyramids of an equal altitude, what equals the sum of their volumes?

15. How may the relative magnitude of prisms which have equal altitudes be found?

16. If a and b are two prisms having equal altitudes and



PRISMS AND PYRAMIDS.

the base of a equals $\frac{5}{7}$ of the base of b, what is the relation of a to b?

17. What is a prism? When are prisms equivalent?

18. How may the volume of a prism be found? of a pyramid?

19. Draw a rectangular prism equivalent to a pyramid having a base of 6 sq. in. and an altitude of $4\frac{1}{2}''$.

20. What are the dimensions of a rectangular solid equivalent to a triangular pyramid having a base of 12 sq. in. and an altitude of 15″?

21. What are the dimensions of a rectangular solid equivalent to a cone whose base equals 12 sq. in. and altitude 15″?

22. The largest cone that can be turned out of a cubic foot of wood equals what part of the cubic foot?

23. How many cubic inches in the largest cone that can be turned out of a cubic foot of wood?

24. The largest pyramid that can be cut out of a square prism equals what part of the prism?

25. What is the volume of a pyramid whose base is 84 sq. in. and whose altitude is 72″?

26. What is the volume of a cone if its altitude is 35 ft. and the diameter of its base is 15 ft.?

27. What equals the volume of a conical pyramid if its altitude is a and the diameter of its base is b?

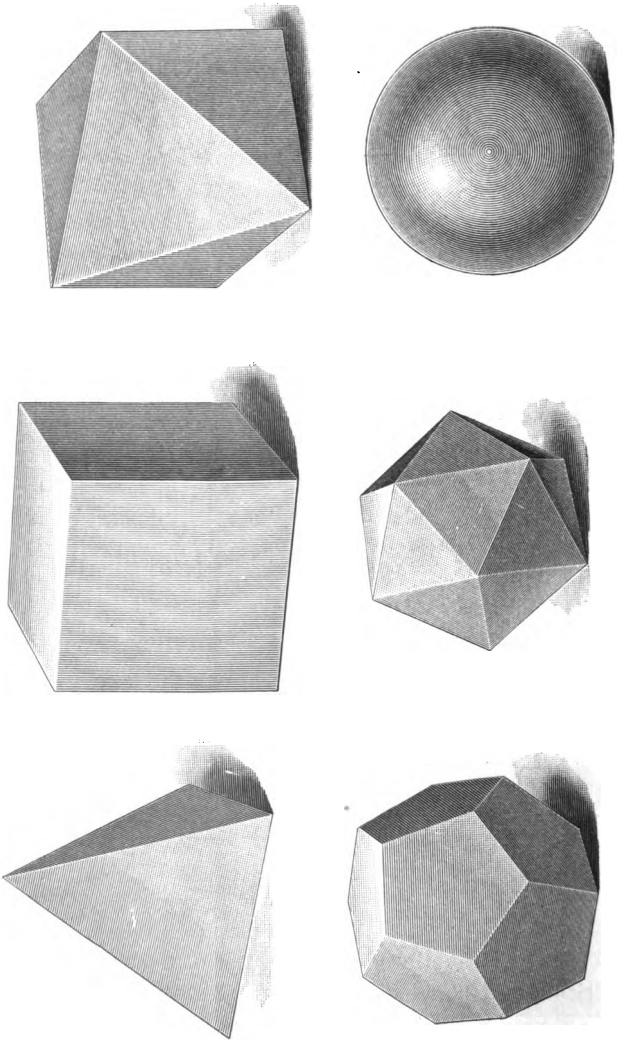
$$\frac{a \cdot 11 \cdot b^2}{3 \cdot 14} = ?$$

28. What is the volume of a cone if the diameter of its base is 7″ and its altitude 9″?

29. What is the volume of a cone if the diameter of its base is 29″ and its altitude 28″?

30. If the circumference of the base of a cone is 54″ and its altitude is 12″, what is its volume?

31. How many yards of canvas $\frac{3}{4}$ yd. wide are required to make a conical tent, 7 ft. slant height, and 8 ft. in diameter?



REGULAR POLYHEDRONS.

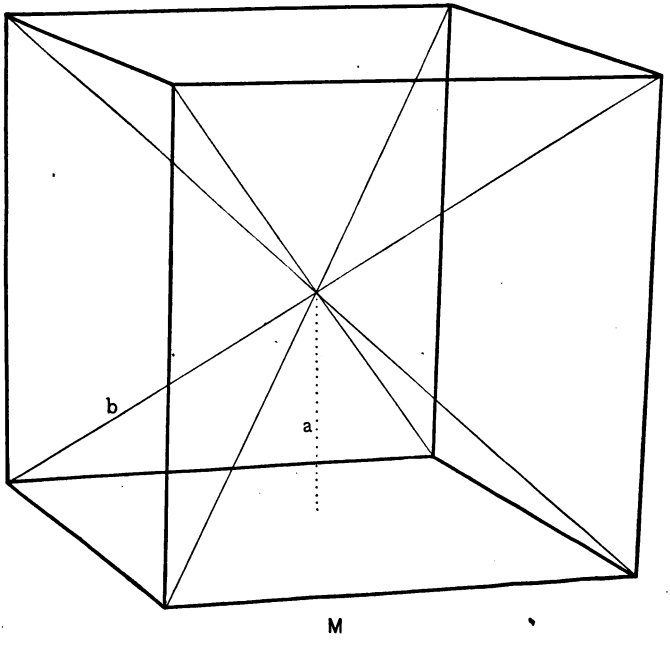
32. What equals the volume of x pyramids of equal altitude, supposing the sum of their bases to be 30 sq. in. and the altitude 8″?

33. How can the area of the base of a pyramid be found if the number of units in its volume and the number of units in its altitude are given? How can the volume of a pyramid be found if the number of units in its altitude and the number of units in its base are given?

Regular Polyhedrons. — 1. In what are the polyhedrons on page 200 alike?

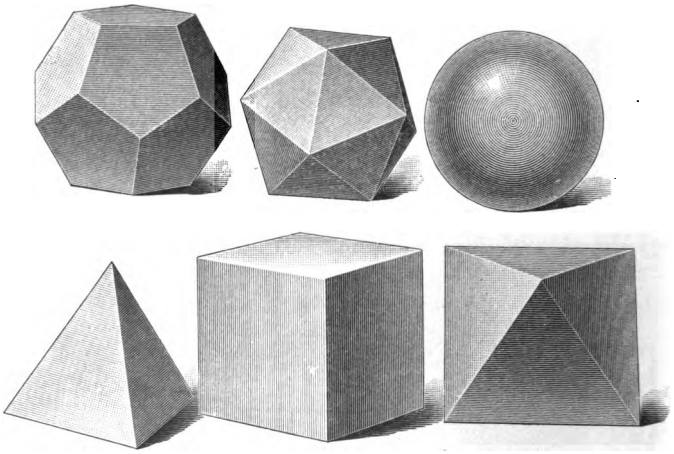
2. Describe the greater radius of each. What is true of the greater radii of each?

3. Describe the less radius of each. What is true of the less radii of each respectively?



4. In the regular polyhedron M, a is a less radius and b a greater radius. Show the greater and less radii of this room.

5. Are all of the greater radii of the room equal? Are all of the less radii equal? Look carefully. Is it the



REGULAR POLYHEDRONS.

same distance from the center of the room to each of the four walls as to the ceiling and to the floor?

6. What is true of the greater and less radii respectively of a regular polyhedron?

7. What is a regular polyhedron?

A regular polyhedron is a solid whose greater and less radii are respectively equal.

8. In which solid is there the greatest difference in the length of the greater and less radii? In which are all the radii equal?

9. What things are respectively equal in a regular polygon? In a regular polyhedron?

10. Into what equal solids can a regular polyhedron be separated?

1. Into how many equal pyramids can a cube be separated?

2. In this cube, find the edges and the base of each of the six pyramids into which the cube may be separated.

3. Find the edges and the base of each of the six pyramids into which this room may be separated.

4. What is the name of the altitude of the pyramids into which a regular polyhedron may be separated?

5. What is the number of sq. in. in the sum of the bases of the six equal pyramids into which a 12″ cube may be separated? What is the altitude of the pyramids in a 12″ cube? The number of units in its volume equals how many times the number of units in the base? By what must the number of units in the surface of a cube be multiplied to equal the number of units in the volume of the pyramids to which it is equal?

6. The number of units in the volume of any regular solid equals the product of what numbers?

7. If a sphere is a regular polyhedron, of what is it composed?

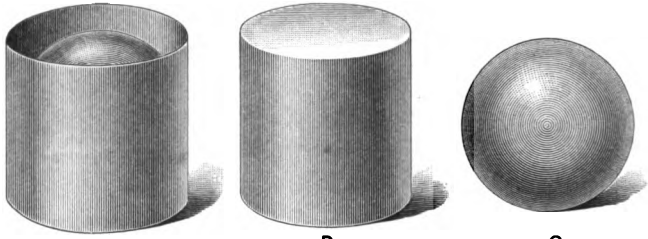
8. What equals the number of units in the volume of a sphere? Is it necessary to use the term *less* radius in giving a rule for finding the volume of a sphere? Why not?

9. Review the work on regular polygons.

Before arriving at an easy method of finding the volume of a sphere, one must know how to find its surface.

The altitude and the diameter of the base of the cylinder *D* are each equal to the diameter of the sphere *O*. See page 204.

If a rectangle equal to the lateral surface of the cylinder D be placed about the sphere O, it could, by great skill in cutting and rearranging, be made to cover exactly the sphere O.



1. Cut a rectangle equal to the lateral surface of a cylinder whose altitude and diameter are each 1 in.

2. Cut a rectangle equivalent to the surface of a sphere 1 in. in diameter.

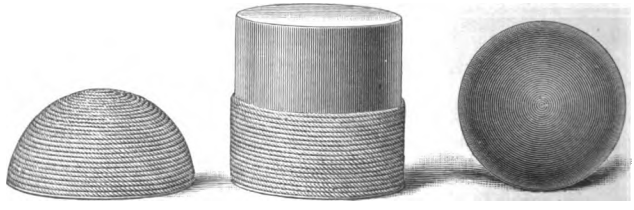
3. Cut a rectangle equivalent to the surface of a sphere $\frac{1}{2}$ in. in diameter.

4. The surface of the sphere $\frac{1}{2}$ in. in diameter equals what part of the surface of the sphere 1 in. in diameter?

5. Draw a rectangle 2 in. wide equivalent to the surface of a sphere 2 in. in diameter.

6. What is the ratio of the surface of a sphere 2 in. in diameter to the surface of a sphere 1 in. in diameter? to the surface of a sphere 4 in. in diameter?

7. If a is the diameter of a sphere, what may be the dimensions of a rectangle equal to the surface of the sphere?



The altitude and the diameter of the cylinder are each equal to the diameter of the sphere and of the hemisphere. The length of the cord which covers the curved surface of the hemisphere equals the length of the cord which covers one-half of the lateral surface of the cylinder. Compare the lateral surface of the cylinder with the surface of the sphere. See page 204.

8. Observe spheres and cylinders and compare surfaces.

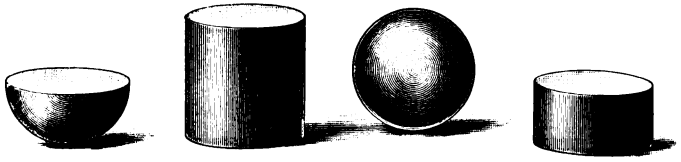
9. What are the dimensions of a rectangle equivalent to a circle 1 in. in diameter? 2 in. in diameter?

10. What are the dimensions of a rectangle equivalent to a great circle of a sphere 1 in. in diameter?

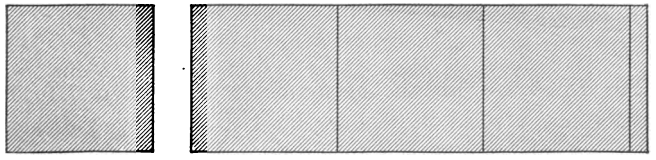
11. What is the ratio of a great circle of a sphere to the surface of the sphere?

12. A great circle of the earth equals what part of its surface?

13. Discover the ratios of the surfaces of the solids in the following cut:



14. Draw a rectangle equivalent to the surface of a sphere 1 in. in diameter. Make rectangle 1 diameter wide.



15. Express in diameters the dimensions of a rectangle equivalent to the surface of any sphere. What is the ratio of this rectangle to the square of the diameter? $3\frac{1}{7}$ or $\frac{22}{7}$ is the ratio of the surface of a sphere to what?

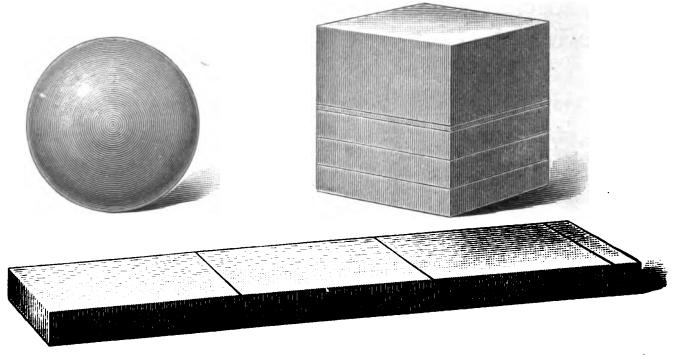
16. What is the name of the equal solids into which any regular solid may be separated? By what must the number of units in the surface of a regular solid be multiplied to equal the number of units in the volume of the regular solid?

17. Give several different methods of finding the volume of a sphere.

18. Draw a rectangular solid equal to the volume of a sphere 1 in. in diameter.

What are the dimensions of the solid expressed in inches? in diameters?

19. What are the dimensions of a rectangular solid equal to a sphere 2 in. in diameter. Express the dimensions in inches; in diameters.



20. Draw the cube of the diameter of a sphere 1 in. in diameter. A solid $3\frac{1}{7}$ d. long by 1 d. wide by $\frac{1}{8}$ d. high equals what part of the cube of its diameter?

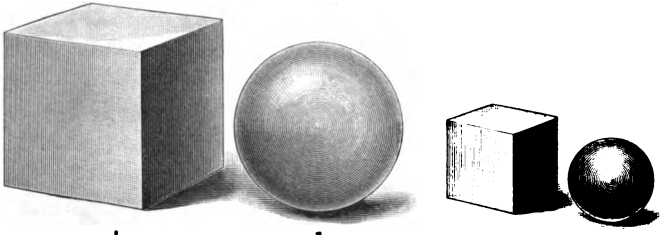
21. Any sphere equals what part of the cube of its diameter?

22. What is the volume of spheres having the following diameters: 14″; $7\frac{1}{2}$″; 42 ft.; .7 yd.; 2.9 ft.; $8\frac{3}{7}$″?

$\frac{11 \cdot 14^3}{21}$ = the number of cubic inches in a sphere 14 in. in diameter.

23. Measure the diameters of many spherical bodies — baseballs, croquet balls, drawing models, etc., and find $\frac{11}{21}$ of the cube of their diameters.

1. What is the ratio of a to b? c equals what part of a? d equals what part of b? Then what is the ratio of c to



d? Spheres are to each other as the cubes of what? Why?

2. What is the ratio of a sphere 1 in. in diameter to a sphere 2 in. in diameter? Why?

3. What is the ratio of a sphere 2 in. in diameter to a sphere 4 in. in diameter? to a sphere 3 in. in diameter? to a sphere 1 in. in diameter? to a sphere $\frac{1}{2}$ in. in diameter?

4. What is the ratio of a sphere $\frac{1}{2}$ ft. in diameter to a sphere $\frac{3}{4}$ ft. in diameter?

5. What is the ratio of a sphere 1 in. in diameter to a 2-in. cube? *Ans.* $\frac{11}{21}$ of $\frac{1}{8}$ is the ratio of a sphere 1 in. in diameter to a 2-in. cube.

6. Observe cylinders, cones, cubes, spheres, and other solids, and give ratios.

1. What is the volume of the largest sphere that can be turned out of a cubic foot of wood?

2. What is the surface of the largest sphere that can be turned out of a cube 2 ft. long?

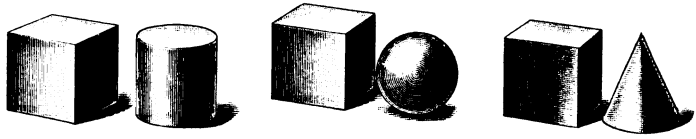
3. What is the entire surface of an 18″ cube? What is the length of its less radius? What is the area of the base of one of its six pyramids? What is the volume of one of the pyramids? What is the volume of the cube?

4. Give a rule for finding the volume of a cube. Give a rule for finding the volume of any regular solid.

5. Compare the rule for finding the area of a regular polygon with the rule for finding the volume of a regular solid.

6. What is the volume of the largest cone that can be turned out of a cubic foot?

7. What are the relations of the largest cylinder, sphere, and cone that can be turned out of equal cubes?



8. Give two rules for finding the volume of a sphere.

9. What is the volume of a cube 6 ft. long? What is its entire surface?

10. The base of a rectangular solid is a square inch. What is the altitude if its volume equals that of a cylinder whose altitude is 1 in. and the diameter of whose base is 1 in.?

The base of a rectangular solid is a square inch. What is the altitude if its volume equals that of a sphere 1 in. in diameter?

The base of a rectangular solid is a square inch. What is the altitude if its volume equals that of a cone 1 in. in altitude and the diameter of whose base is 1 in.?

What is the relative height of the three solids that are respectively equal to the cylinder, the sphere, and the cone?

11. What is the altitude of a rectangular solid whose base is a^2 if the volume of the solid equals a cylinder whose altitude is a and the diameter of whose base is a?

1. What is the volume of a cylinder whose diameter and altitude are each 1? 6? 10? x? What is the volume of a sphere whose diameter is 1? 6? 10? x?

Then what is the ratio of the volume of a sphere to the volume of a cylinder whose diameter and altitude each equal the diameter of the sphere?

2. What is the volume of a cone whose greatest diameter is 1 and whose altitude is 1? whose greatest diameter and altitude are 6? 10? x?

Then what is the ratio of the volume of a cone whose altitude equals the diameter of its base to the volume of a sphere of equal diameter?

3. What part of the volume of a wooden cylinder whose diameter and altitude are equal must be cut away in order to turn from it the largest possible sphere? in order to turn from it the largest possible cone?

4. Describe "the three round bodies" whose relative value is expressed by the numbers 1, 2, 3.

5. What part of a cube equals the largest cylinder contained in it? the largest sphere? the largest cone?

6. What is the entire surface of a cylinder whose diameter is 1 and whose altitude is 1? What is the entire surface of a cylinder whose diameter is 6 and whose altitude is 6? of a cylinder whose diameter and altitude are 10? of a cylinder whose diameter and altitude are x?

7. What is the surface of a sphere whose diameter is 1? 6? 10? x?

8. What is the ratio of the surface of a sphere to the curved surface of a cylinder whose diameter and altitude each equal the diameter of a sphere? What is the ratio

of the surface of the sphere to the entire surface of the cylinder? the reciprocal ratio?

9. What is the ratio of the curved surface of a hemisphere to the plane surface?

10. What is the ratio of the entire surface of a sphere to the entire surface of a hemisphere of equal diameter? the reciprocal ratio?

11. What is the ratio of the curved surface of one-half a hemisphere to the sum of its plane surfaces? What is the ratio of the entire surface of one-fourth of a sphere to the entire surface of a hemisphere? to the entire surface of a sphere?

12. What is the ratio of the entire surface of one-eighth of a sphere to a great circle of that sphere?

13. What is the ratio of the surface of a sphere to the entire surface of a cube whose edge equals the diameter of the sphere?

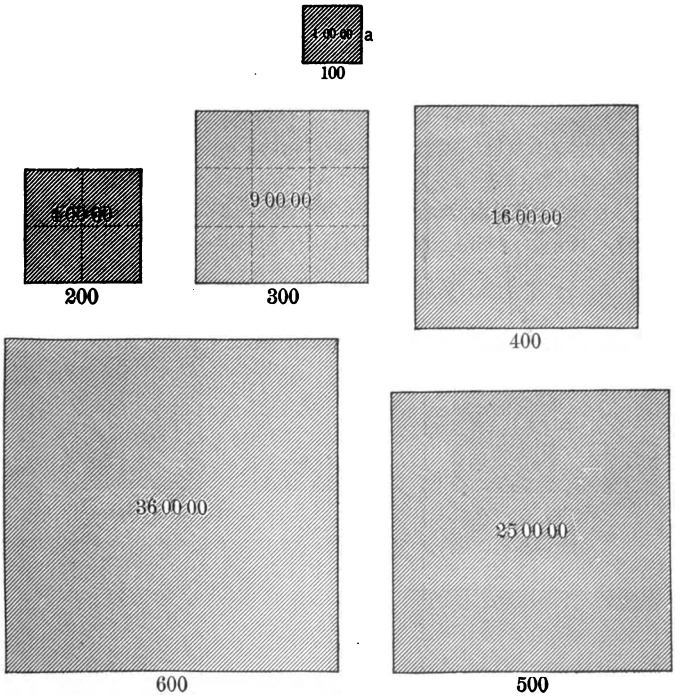
14. What is the ratio of the entire surface of a hemisphere to the entire surface of the smallest square prism that will contain it? What is the ratio of the curved surface of the hemisphere to the surface of such a square prism?

15. What is the slant height of a cone — diameter of base 1 — whose curved surface is double its plane surface? of a cone whose plane surface equals $\frac{1}{4}$ of its curved surface?

16. What is the ratio of the entire surface of each of the cones in the last to the entire surface of a hemisphere whose diameter equals the greatest diameter of the cone?

17. What is the ratio of the curved surfaces of these cones to each other and to the curved surface of the hemisphere?

18. What are the ratios of the curved surfaces of the largest sphere, cylinder and cone that can be cut from equal cubes?



Square Root. — 1. What is the ratio of each unit to each of the others?

2. If 100 is the length of a, what is the length of each of the others?

3. If a is 100^2 (*Read:* If a is the square of 1 hundred), each of the other units is the square of what?

4. What is the ratio of 400^2 to the square of each of the others?

5. What is the ratio of each square to each of the others?

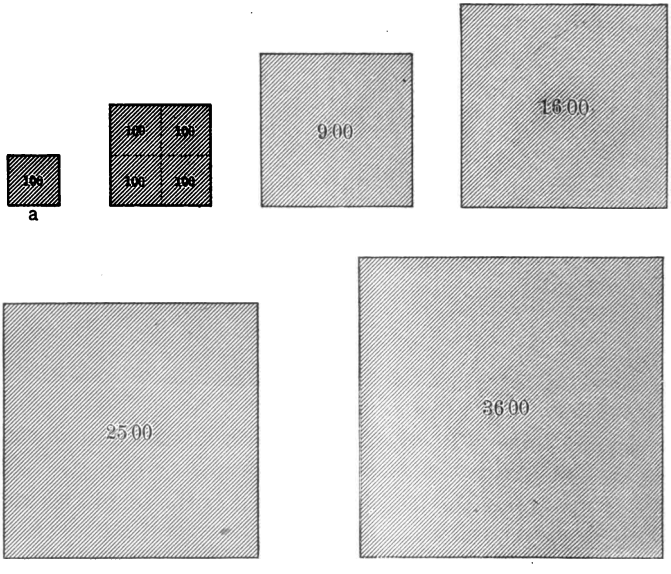
6. What is the ratio of 700^2 to each? of 800^2? of 900^2?

7. 4 is the ratio of the squares of which units between 100 and 900 inclusive? 9 is the ratio of the squares of what? $\frac{1}{4}$ is the ratio of the squares of what? $\frac{1}{9}$? 16? $\frac{1}{16}$?

8. What is the ratio each to each between 100^2 and 900^2 inclusive? What is the ratio of 200^2 to 400^2? to 600^2? to 800^2?

9. What is the ratio of 300^2 to 600^2? to 900^2?

10. What is the ratio of 400^2 to 200^2?



1. If 10 is the length of a, what is the length of each of the other units?

2. If a is 10^2, each of the other squares is the square of what?

3. What is the ratio of 40^2 to the square of each of the others?

4. What is the ratio of 20^2 to the square of each of the others? of 50^2? of 60^2? of 80^2? of 90^2? of 70^2?

5. 4 is the ratio of which squares? 9 is the ratio of which squares? $\frac{1}{4}$ is the ratio of which squares? $\frac{1}{9}$? $\frac{1}{16}$? 16?

6. What is the ratio of 10^2 to each between 10^2 and 90^2 inclusive? of 20^2 to each? of 30^2? of 40^2?

1. What is the ratio of 100^2 to 300^2? of 10^2 to 30^2? of 1^2 to 3^2? of $.1^2$ to $.3^2$?

2. What is the ratio of 100^2 to 400^2? of 10^2 to 40^2? of 1^2 to 4^2? of $.1^2$ to $.4^2$?

3. What is the ratio of 100^2 to 700^2? of 10^2 to 70^2? of 1^2 to 7^2? of $.1^2$ to $.7^2$?

4. What is the ratio of 1^2 to 6^2? to 10^2? to 100^2?

5. What is the ratio of 1^2 to 9^2? to 90^2? to 900^2?

6. What is the ratio of 1^2 to 7^2? to 70^2? to 700^2?

Write the following in figures and practice reading:

1 ten-thousand; 9 ten-thousand; 15 ten-thousand; 71 ten-thousand; 13 ten-thousand; 7 ten-thousand 8 hundred; 67 ten-thousand 15 hundred; 49 ten-thousand 17 hundred 9 tens; 63 ten-thousand 22 hundred 4 tens; 84 ten-thousand 29 hundred 64; 33 ten-thousand 94 hundred 75; 20 ten-thousand 5 hundred 3 tens; 91 ten-thousand 7 hundred 1; 70 ten-thousand 27 hundred 82; 5 ten-thousand 3 hundred 7; 48 ten-thousand 93 hundred 5 tens; 18 ten-thousand 30 hundred; 12 ten-thousand 9; 3 ten-thousand 3; 64 ten-thousand 36 hundred 50; 60 ten-thousand 2 hundred 40; 6 ten-thousand 3 hundred 7; 40 ten-thousand 7 hundred 1; 90 ten-thousand 9 hundred 9; 30 ten-thousand 4 hundred 90; 30 ten-thousand 30 hundred 30; 3 ten-thousand 3 hundred 3; 45 ten-thousand 1 thousand 10.

16 00 00 is 400^2. *Read:* 16 ten-thousand is the square of 4 hundred.

What is the square of 300? of 200? 500? etc.

Review pp. 238, 270, and 300, Speer's "Elementary Arithmetic."

1. Can a square be cut into 2 equal squares? into 5 equal squares? into 9? into 10? into 15?

2. When a square is divided into 4 equal squares, how are its edges divided? if divided into 9 equal squares? etc.

Ex. The edge of a square that is divided into 4 equal squares is divided into halves.

3. What are the different *numbers* of equal squares between 1 and 100 into which any square can be divided?

4. What is the least *number* of equal squares of which a square can be made? Give all the *numbers* of equal squares between 1 and 100 of which perfect squares can be made.

5. What is 10^2? What is the least number of 10^2 (*Read:* squares of 10) that can be placed together to make a perfect square? What are the different numbers of 10^2 between 1 and 100 of which perfect squares may be made?

6. What is 100^2? What is the least *number* of 100^2 that can be placed together to make a perfect square? Give all of the different *numbers* of 100^2 between 1 and 100 that may compose perfect squares.

7. What is the largest square composed of inch squares that can be made of 5 sq. in.? What is the length of this square? 4 is the number of what?

8. What is the largest square composed of 100^2 (*Read:* squares of 100) that can be made of 5 00 00 (5 ten-thousand)? What is the length of this square? Then what is $\sqrt{4\ 00\ 00}$?

9. What is the largest square composed of 10^2 that can be made of 18 00? What is the length of this square? What is the $\sqrt{1600}$?

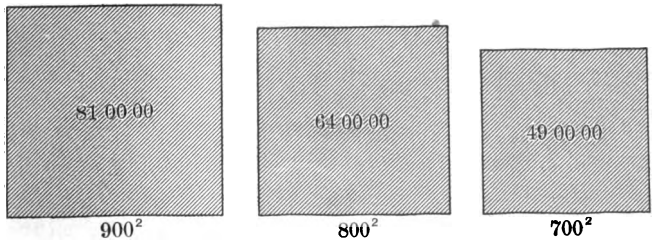
10. What is the largest square composed of 1^2 that can be made of 85? of 34? of 10? of 20? of 47? of 75? of 50? What is the length of each square?

11. What is the largest square composed of 100^2 that can be made of 50 00 00? of 75 23 72 (*Read:* 75 ten-thousand 23 hundred 72)? of 10 00 00? of 1 00 00? of 35 73 22? of 99 99 99? of 8 27 31? of 15 12 43? of 22 22 22? What is the length of each square?

12. What is the largest square that can be made of 10^2 (squares of 10) contained in 4 83? in 38 33? in 10 00? in 27 27? in 45 45? What is the length of each square?

13. What is the largest square that can be made of $.1^2$ contained in .69? in 5.25? in 37.33? What is the length of each square?

14. What is the largest square of hundredths in .00 38? in .12 38? in 4.37 62?



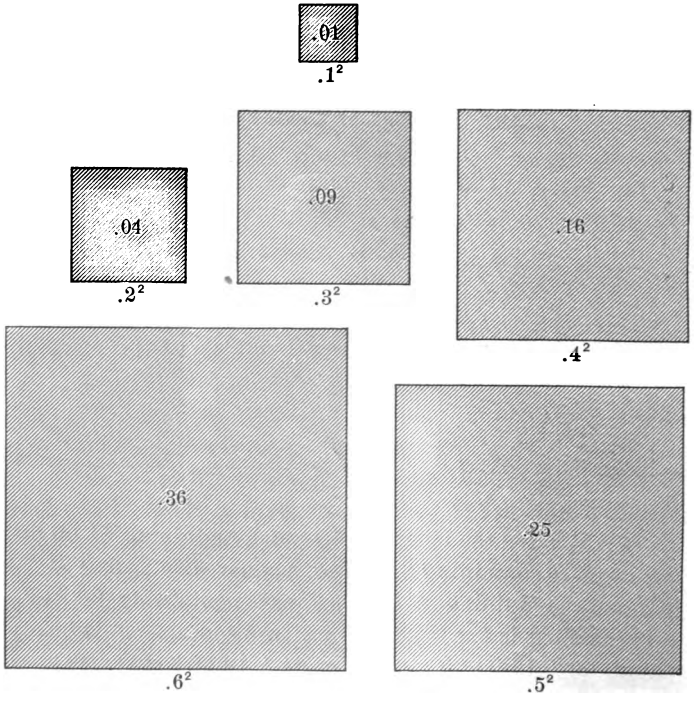
1. What is the length of a square containing 81 00 00? of a square containing 81 00? of a square containing 81? of a square containing .81? of a square containing .00 81?

2. What is the length of a square containing 64 00 00? 49 00 00? 16 00 00? 64 00? .64? 49 00? 49? .49?

3. What is the length of a square containing 25 00 00 ? 25 00 ? 25 ? .25 ? .00 25 ? .00 00 25 ?

Write and memorize the squares of all the different numbers of hundreds from 100 to 900 inclusive. Image them as divided into 100^2. *Ex.*: In the 300^2 I see 9 squares of 100, or 9 00 00 (*Read:* 9 ten-thousand). Write and memorize the squares of all the different numbers of tens from 1 ten to 90 inclusive. Image them as divided into 10^2. How many hundred in each ?

Write and memorize the squares of all the different numbers of tenths from .1 to .9 inclusive. How many $.1^2$ in each ? How many hundredths in each ?



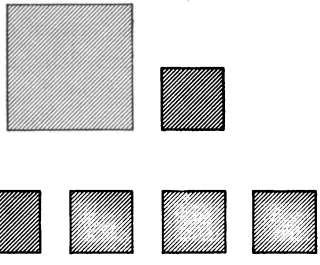
$100^2 = 1\ 00\ 00$	$10^2 = 1\ 00$	$1^2 = 1$	$.1^2 = .01.$
$200^2 = 4\ 00\ 00$	$20^2 = 4\ 00$	$2^2 = 4$	$.2^2 = .04.$
$300^2 = 9\ 00\ 00$	$30^2 = 9\ 00$	$3^2 = 9$	$.3^2 = .09.$
$400^2 = 16\ 00\ 00$	$40^2 = 16\ 00$	$4^2 = 16$	$.4^2 = .16.$
$500^2 = 25\ 00\ 00$	$50^2 = 25\ 00$	$5^2 = 25$	$.5^2 = .25.$
$600^2 = 36\ 00\ 00$	$60^2 = 36\ 00$	$6^2 = 36$	$.6^2 = .36.$
$700^2 = 49\ 00\ 00$	$70^2 = 49\ 00$	$7^2 = 49$	$.7^2 = .49.$
$800^2 = 64\ 00\ 00$	$80^2 = 64\ 00$	$8^2 = 64$	$.8^2 = .64.$
$900^2 = 81\ 00\ 00$	$90^2 = 81\ 00$	$9^2 = 81$	$.9^2 = .81.$

$100^2 = 1\ 00\ 00.$ $400^2 = 16\ 00\ 00.$

$10^2 = 1\ 00.$ $40^2 = 16\ 00.$

$1^2 = 1.$ $.4^2 = .16.$

Write the above from memory. Begin at the top and write to the bottom; reverse. Begin at left and write to right; reverse. Give these equations at random.



1. What is the largest square that can be made of 5 ($5 \cdot 1^2$)? of 5 00 ($5 \cdot 10^2$)? of 5 00 00 ($5 \cdot 100^2$)?

2. What is the edge or root of the largest square in 5 composed of 1^2? of the largest square in 5 00 composed of 10^2? the largest in 5 00 00 composed of 100^2? the largest in .05 composed of $.1^2$? the largest in .00 05 composed of $.01^2$?

3. What is the largest square of ones in 79? of tens in 79 00? of hundreds in 79 00 00? of tenths in .79? of hundredths in .00 79?

4. What is the largest square of tens in 5 25? its root? remainder? *Ans.* 400 is the largest square of tens in 525; its root is 20; the remainder is 125.

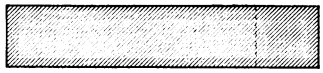
5. What is the largest square of tens, its root, and the remainder in each of the following: 6 24; 11 62; 16 79; 75 25?

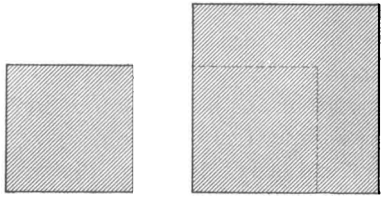
6. What is the largest square of hundreds, its root, and the remainder in each of the following: 6 24 00; 11 62 00; 16 79 00; 92 16 00; 75 25 00?

7. What is the largest square of hundredths, its root, and the remainder in .11 23? *Ans.* .09 is the largest square of hundredths, .3 is its root, and .02 23 is the remainder.

8. What is the largest square of hundredths, its root, and the remainder in each of the following: .06 24; .11 62; .16 79; .92 16; .75 25; .40; .50?

9. What is the largest square of ones, its root, and the remainder, expressed in hundredths, in each of the following: 10; $1\frac{1}{2}$; 12.38; 6.7; $3\frac{2}{3}$; 4.2? *Ex.* 9 is the largest square of ones in 10; its root is 3, and the remainder is 1.00 ($\frac{100}{100}$).





10. What is the largest square of hundredths in .83? in .80? in .8? in .66$\frac{2}{3}$ or $\frac{2}{3}$? in .50 or .5? in .75 or $\frac{3}{4}$? in .12? in .12$\frac{1}{2}$? in .125? in $\frac{1}{8}$? in .375 or $\frac{3}{8}$?

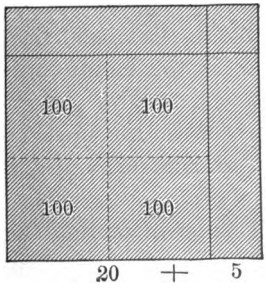
11. In the above, what is the edge of each square and what is the remainder?

12. What is the largest square of ten-thousandths in .0038? in .0082? in .004? in .006? in .0016? in .0076? in $\frac{1}{8}$? in $\frac{2}{8}$? in $\frac{1}{8}$?

13. What is the largest square of hundreds in 576? What is the length of the edge of the square 400? To how many sides of the square 400 must the 176 be added to preserve the form of a square? What is the length of the sum of two edges of the square 400? If the area to be added is 176, what is the approximate width of the additions?

1. Show by a drawing the form of the difference of two squares. Straighten the *L*-shaped difference into a rectangle without altering its width. The length of the rectangle equals how much more than twice the length of the smaller square?

2. What is the largest square of hundreds, its root, and the remainder in 625? Conceive the remainder as a rectangle. What is its area? Approximate its length. If its length were 40, what would be its approximate width? If the length of this rectangle were 45 and its width 5, what would its area be? Then 225 is the difference between what square and 20^2?

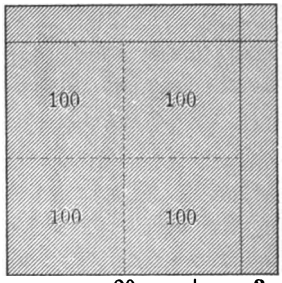


$$\begin{array}{r|l} & 625\ (25 \\ & 4 \\ \hline 45 & 225 \\ & 225 \\ \hline \end{array}$$

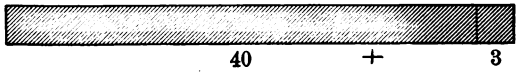
What is, therefore, the $\sqrt{625}$? of 625 half-inch squares? of 6.25? of 625 equal squares of any kind?

What is the square root of 625 sq. ft.? of 625 sq. yd.? of 625 sq. rd.? of 6.25?

3. What is the edge of the largest square of hundreds in 529?



$$\begin{array}{r|l} 529 & 23 \\ 400 & \\ \hline 43 \mid 129 & \\ \quad 129 & \\ \hline \end{array}$$



What is the largest square of hundreds in 500? Show the 400 in the diagram.

What is the length of two sides of the square to which the 129 is to be added?

If 129 is the number of units in the area of the addition, and 40 is nearly the number in the length, what is the number in the width?

Then 129 is the difference between the square of 20 and the square of what? What is, therefore, $\sqrt{529}$? of 529 sq. ft.? of 5.29? of .0529?

Mentally picture the work done in finding the edge of the square 529.

What is the edge of the square 484?

4. What is the length of the edge of the square of the hundreds in the square 1764?

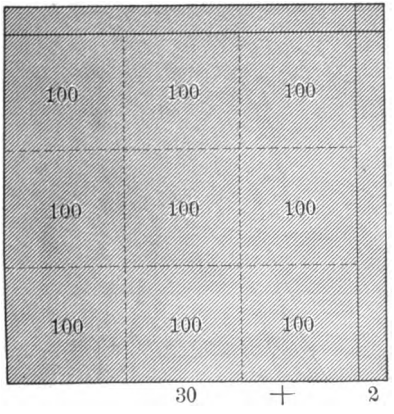
What is the length of the square 1600?

What is the sum of two edges of the square 1600 ?

If 80 is nearly the number of units in the length of a rectangle and 164 is the number of units in its area, what do you think is its width ?

What is the length of the square 1764 ? What is the square root of 1764 ?

5. What is the square root of 1024 ?



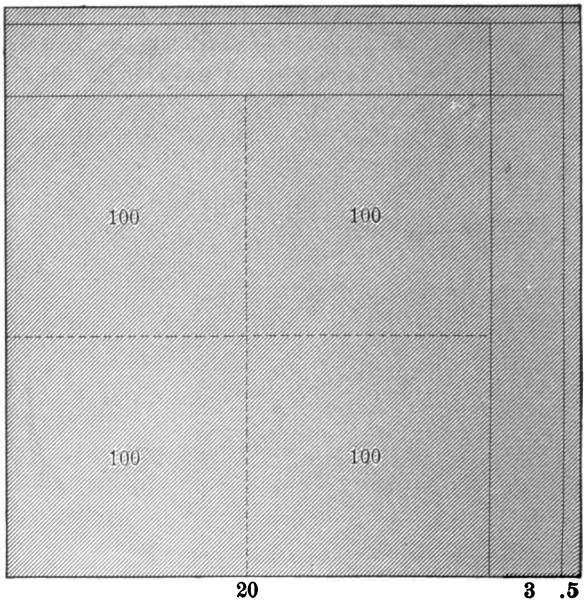
$$\begin{array}{r} 10\ 24\ (32 \\ 9 \\ \hline 62 \mid 124 \\ \quad\ 124 \\ \hline \end{array}$$

6. Find the square root of 53 29 ; 8 41 ; 72 25 ; 98 01 ; 10.89 ; 14.44 ; .28 09 ; .29 16 ; .20 25 ; .8649 ; .72 8.

7. What is the edge of the square of the hundreds in 552 ?

$$\begin{array}{r} 5\ 52\ (23 \\ 4 \\ \hline 43 \mid 152 \\ \quad\ 129 \\ \hline \quad\ 23 \end{array}$$

8. What is the edge of the square 5 52.25 ?



1. What is the square root of 552.25?

What is the largest square of hundreds in 552? What is its root? What is the remainder?

```
552.25 (23.5
   4
43 | 152
     129
46.5 | 23.25
       23.25
```

What is the length of the sum of two sides of the square?

If 152 is the number of units in the sum of the rectangles to be added, and 40 is nearly the number in the sum of their lengths, what is the number in the width?

If 3 is the number in the width, what is the entire number in the length? Why add 3 to 40 before multiplying by 3?

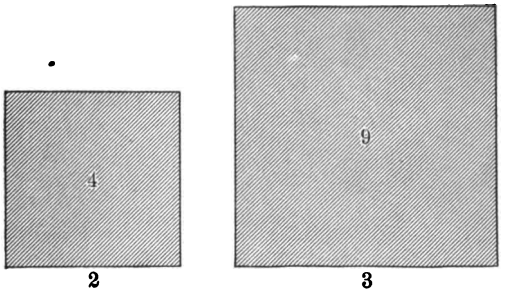
What is now the number of tenths in the sum of two sides of the square?

If 2325 is the number of hundredths in the sum of the rectangles to be added, and 460 is nearly the number of tenths in the sum of their lengths, what is the number of tenths in the width?

465 tenths is the sum of what? .5 of 46.5 equals what? $\therefore 552.25 = ?$

What is the square root of 5.52 25? of 5 52 25?

2. Find $\sqrt{11\ 90\ 25}$; $\sqrt{1\ 65.49}$.



1. What is the ratio of 4 to 9? What is the ratio of an edge of the 4 to an edge of the 9?

Then does $\sqrt{\frac{4}{9}} = \frac{2}{3}$?

2. What is $\sqrt{\frac{9}{16}}$; $\sqrt{\frac{25}{36}}$; $\sqrt{\frac{81}{100}}$; $\sqrt{\frac{64}{121}}$; $\sqrt{\frac{529}{1024}}$; $\sqrt{\frac{625}{3204}}$?

1. Divide a square into tenths and make the equal parts squares if you can.

Can a square be divided into tenths making each tenth a square?

Is .1 by .1 of a square square?

What is the name of such a square?

What is the length of the square .01?

A square .1 by .1 equals what part of 1^2?

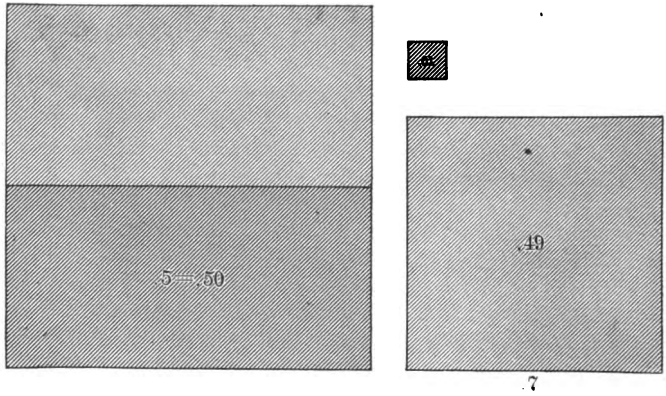
A square .2 by .2 equals how many hundredths? .7 by .7? .5 by .5?

Into what must tenths be changed before the square root can be found?

2. What is the square root of .5?

.5 equals how many hundredths?

What is the largest square of hundredths in .50? What is the square root of .49? of .50?



What is the square root of .7? of .15? of .3? of 1.1? of 3.3?

3. Can you find the square root of .007?

Why not? .007 equals how many ten-thousandths?

What is the largest square of ten-thousandths in .0070?

What is the root or edge of .00 64?

Ans. $\sqrt{.00\ 7} = \sqrt{.00\ 70} = .08 +$.

4. Show the places that the squares of hundreds, tens, ones, tenths, hundredths, and thousandths respectively occupy.

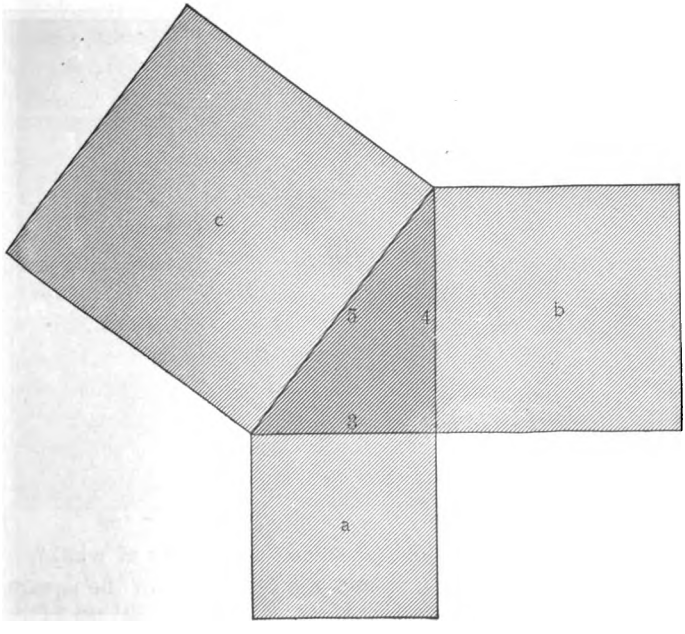
5. How many figures in the square root of each of the following: 9 25; 84 32; 7 85.95; 8 76.5; 42.73 0?

6. What is the largest square of ones in 5? of hundredths in .05? of ten-thousandths in .0005? What is the approximate square root of each?

7. What is the square root of 2 carried to three decimal places?

$$
\begin{array}{r|l}
 & 2.00\ 00\ 00\ (1.414 \\
 & 1 \\
\hline
24 & 1\ 00 \\
 & \quad 96 \\
\hline
281 & 400 \\
 & \quad 281 \\
\hline
2824 & 11900 \\
 & \quad 11296 \\
\hline
\end{array}
$$

8. Carry each to two decimal places:
$\sqrt{3}$; $\sqrt{23}$; $\sqrt{3\frac{1}{4}}$; $\sqrt{7\frac{1}{8}}$; $\sqrt{.05\frac{1}{2}}$.



1. Review p. 166.

2. Show me the squares a, b, c.

3. What does the sum of the squares a and b equal?

4. The sum of a and what equals c? The sum of b and what equals c?

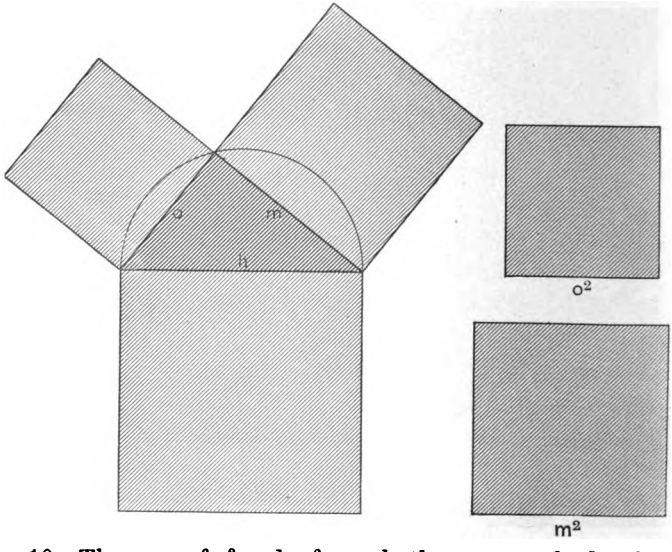
5. c is how much larger than b?

6. If c and a are given, how may the edge of b be found?

7. If a and b are given, how may c be found? the edge of c?

8. If an edge of a and of b is given, how may the edge of c be found?

9. The square of the hypothenuse of a right triangle equals the sum of what?



10. The sum of o^2 and m^2 equals the square of what?

11. The square of h is how much more than the square of o?

12. $\sqrt{o^2 + m^2}$ = what?

13. $\sqrt{h^2 - m^2}$ = what?

14. If h and o are given, how may m be found?

1. How long a rafter is required in building a house 24 ft. wide if the apex of the roof is above the middle line of the house and is 9 ft. higher than the eaves?

2. How long is the string of a kite that is $\frac{1}{4}$ mi. above the earth and directly above a point $\frac{1}{2}$ mi. away from the flyer?

3. What is the slant height of a cone whose altitude is 4 in. and the diameter of whose base is 2 in.?

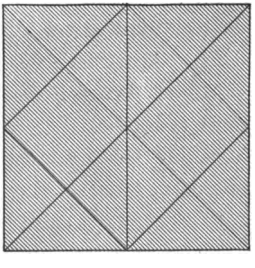
4. How many rods is it from one corner of a square 40-acre field to the opposite corner? How much does a pedestrian save in walking "across lots" when going from one corner to the other?

5. How long is the diagonal of each wall of the schoolroom? of the ceiling?

6. What is the ratio of the diagonal of any square to its side? of the side to the diagonal?

7. Show that $\sqrt{\frac{1}{2}} = \frac{1}{2}\sqrt{2}$ by inspecting the diagram; by calculating each.

8. What is the diagonal of a square 10-acre field?



9. Make a short rule for finding the diagonal of any square from its side. For finding a side from a diagonal. If a side is s, what is the diagonal? If the diagonal is d, what is the side?

Ans. $s\sqrt{2}$; $\frac{d}{2}\sqrt{2}$.

10. What is the diagonal of a rectangle 3 by 4? 6 by 8? 12 by 16? If the altitude of a rectangle equals $\frac{3}{4}$ of the

base, what is the ratio of the diagonal to the base? to the altitude?

11. Find the diagonals of the different faces of a rectangular prism $3 \times 5 \times 8$. Find the diagonal of the prism itself.

12. What is the distance from the upper N. E. corner of the schoolroom to the lower S. W. corner?

13. What is the ratio of the diagonal of a cube to each of its edges?

14. Calculate the lateral sides of an isosceles triangle whose altitude is 8 in. and whose base is 5 in.

15. What is the slant height of the largest cone that can be cut from a cubic foot?

16. Find the slant height of the largest cone that can be cut from a cube whose edge is a.

17. What is the slant height of a square pyramid the area of whose base is 4 sq. in. and whose altitude is 4 in.?

18. What is the length of each edge of such a pyramid?

19. What is the length of a ladder that just reaches a window 40 ft. from the ground, the foot of the ladder being 18 ft. distant from the wall?

20. What is the area of a square whose diagonal is d?

21. What is the diagonal of a square whose area is a?

22. Show that $\sqrt{a} \cdot \sqrt{2} = \sqrt{2a}$ by the last diagram.

23. If the diagonal of a rectangle is d and its altitude a, what is its base? *Ans.* $b = \sqrt{d^2 - a^2}$.

24. If the hypothenuse of a right triangle is h and its base is b, what is the perpendicular?

Ans. $p = \sqrt{h^2 - b^2}$.

25. A is 25 mi. east and B is 30 mi. south of C. How far apart are A and B?

26. What is an edge of the square that equals the difference between 8^2 and 5^2?

27. A house is 32 ft. wide. The roof measures 46 ft. from eave to eave. How high above the eaves is the highest point of the roof?

28. How far is it on a level through a hole 100 ft. deep but 1450 ft. long on a slant from bottom to top?

29. If the side of an equilateral triangle is 1, what is its altitude? If the side is s, what is the altitude?

30. If the side of a rhombus is 1 and its shorter diagonal is 1, what is the longer diagonal?

31. What is the diagonal of a regular hexagon whose side is 1? whose side is s? *Ans.* $s \cdot \sqrt{3}$.

32. What is the less radius of a regular hexagon whose side is 1? whose side is s?

33. What is the altitude of an isosceles triangle whose base is 60 and whose equal sides are each 70?

34. What is the shorter diagonal of a rhombus whose longer diagonal is 60 and whose sides are 40?

35. What is the longer diagonal of a rhombus whose shorter diagonal is 80 and whose sides are 100?

36. One side of a rectangle is 60 ft.; its diagonal is 75 ft. What is the other side of the rectangle?

37. What is the ratio of the greater radius of a square to an edge? to the less radius?

Of the greater radius of a cube to an edge? of the less radius?

38. Let r and s represent the areas of two squares. Represent the ratio of their edges. *Ans.* $\frac{\sqrt{r}}{\sqrt{s}}$.

1. Draw two equal or unequal lines. Draw a line equal to their sum. Draw the square of each of these three lines. Is the square of the sum of the edges of two squares equal to the sum of the squares? Test this principle with other lines.

2. What must be added to the sum of two squares to equal the square of the sum of their edges? Show by diagram.

3. What must be added to the sum of 7^2 and 5^2 to make $(7+5)^2$? What equals $(8+7)^2$? $(2+6)^2$? $(11+4)^2$? $(6+9)^2$? $(5+10)^2$? $(3+12)^2$? $(a+b)^2$? $(x+y)^2$?

4. What equals $(x+1)^2$? $(a+1)^2$? $(4+1)^2$? $(8+1)^2$?

5. What equals the difference between two squares whose roots are consecutive numbers?

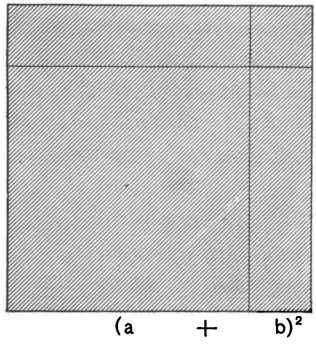
6. $6^2 = 5^2 + 11$. $8^2 = 7^2 +$ what? $9^2 = 8^2 +$ what? $10^2 = 9^2 +$ what? $(x^2 + 1^2) = x^2 +$ what?

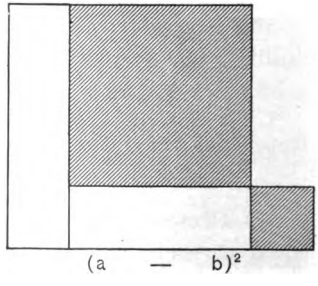
7. $21^2 = 400 +$ what? $51^2 = 2500 +$ what? $91^2 =$ what? $41^2 =$ what?

8. What equals 101^2? 201^2? 301^2? 99^2? 19^2? 29^2? 39^2? $(x-1)^2$?

9. Does the difference between two squares equal the square of the difference between their roots?

10. Make a diagram to show how much must be taken from the sum of two squares to leave a remainder that is equal to the square of the difference between their roots.





11. What must be taken from $a^2 + b^2$ to leave $(a-b)^2$? added to $(a-b)^2$ to equal $a^2 + b^2$?

12. What equals the difference between $a^2 + b^2$ and $a^2 - b^2$?

13. How much must be added to $(a - b)^2$ to make $(a + b)^2$?

14. What equals the square of $(7 - 4)$? of $(8 - 5)$? of $(12 - 9)$? of $(13 - 10)$?

15. Cut a figure equal to the difference between two squares. Cut an equivalent rectangle whose base equals the sum of the square roots or edges. What is the altitude of this rectangle?

16. Indicate the difference between a^2 and b^2. Indicate area of the rectangle in question 15. Make an equation.

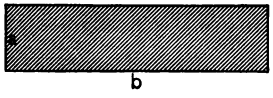
Ans. $a^2 - b^2 = (a + b) \cdot (a - b)$.

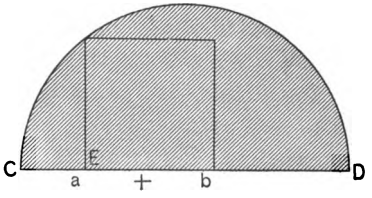
17. What equals $25^2 - 5^2$? $32^2 - 8^2$? $18^2 - 12^2$? $72^2 - 28^2$?

18. What must be added to 3^2 to equal 13^2? to equal 27^2? 12^2? 33^2? 7^2?

19. What equals the side of a square equivalent to a rectangle whose base is b and altitude a?

To find the side of such a square, make CD equal the sum of a and b. Describe a semicircumference on CD.





From E erect a perpendicular terminating in the curve. This is a side of such a square.

20. What is the edge of a square field equal to a rectangular field 10 rd. by 16 rd.?

An arc is any part of the circumference of a circle.

A chord is a straight line joining any two points in the circumference of a circle.

An ordinate is half a chord; the distance from any point in the diameter of a circle to a point on the circumference directly opposite is an ordinate.

21. What is the value of an ordinate the foot of which is 5 ft. from the center of a circle whose radius is 10 ft.? In this circle, what is the value of a chord 4 ft. from the center? 3 ft. from the center? 1 ft.?

22. In a circle whose radius is 6 in., what is the value of a chord that is 2 in. from the center? of a chord whose greatest distance from the arc which it subtends is 2 in.? 1 in.?

23. In a circle whose radius is 5 in., what is the value of a chord whose greatest distance from the arc which it subtends is 1 in.? 2 in.? 3 in.? 4 in.?

24. In a circle whose radius is r, what is the value of an ordinate whose foot is x in. from the circumference?

25. In a circle whose radius is 10 ft., calculate the distance of the circumference from points on the diameter at intervals of 2 ft. Calculate the departure of the circumference from a straight line touching the circle and parallel to the diameter.

26. What is the ratio of the altitude of an equilateral triangle to a side? If the side is 1, what is the altitude? If the altitude is 1, what is a side? If a side is 1, what is the area? if the side is s? If the area is 1, what is the square of a side? What is a side?

Ans. to last. $s = \frac{\sqrt{100}}{\sqrt{43}}$.

27. If the area of an equilateral triangle is 1 acre, what is the length of each side? the altitude?

Ans. $\frac{\sqrt{100\ 160}}{\sqrt{43}}$ = the number of rods in side. $2\frac{\sqrt{43}}{\sqrt{100}}$ = number of rods in altitude.

28. If the side of a hexagon is 1, what is the less

radius? If the less radius is 1, what is a side? If a side is 1, what is the area? If the area is 1, what is a side?

Ans. $s = \frac{\sqrt{100}}{\sqrt{6.43}}$.

What is the ratio of the less radius to a diagonal? of a greater radius to a diagonal? of a diagonal to a side?

29. What is the ratio of a hexagon to an equilateral triangle if the ratio of the side of the hexagon to the side of the equilateral triangle is $\frac{1}{3}$?

30. What is the length of the smallest square from which a regular octagon may be cut whose side is 1? What is the area of this square? What is the area of each of the four right triangles that must be subtracted from the square to make the regular octagon? Then what is the area of the regular octagon whose side is 1? What is the ratio of the area of the regular octagon to the area of the square? of the area of the square to the area of the regular octagon?

31. If the side of the square is 1, what is the side of the largest regular octagon that may be made from it?

Ans. $s = \frac{100}{242}$.

If a falling body moves 16 ft. the first second, 3 times 16 ft. the second second, 5 times 16 ft. the third second, and 7 times 16 ft. the fourth second, how far does it fall in 4 sec.? How far does it fall in 5 sec.? in 8 sec.? in 50 sec.? in 1 min.? in x sec.?

At this rate, how long does it take a body to fall 144 ft.? 576 ft.? 1000 ft.? 1 mi.? x ft.?

What do you notice about the sum of all the odd numbers from 1 to 19 inclusive? About the sum of all the odd numbers from 1 to any other odd number?

If I cut a hole an inch square in a cardboard and hold it 1 ft. from an electric light, how much of a surface parallel

to the card and 2 ft. from the light will be illuminated? 3 ft. from the light? 7 ft.? 10 ft.? x ft.? Then how does the intensity of the light upon 1 sq. in. of surface in any of these cases compare with that upon 1 sq. in. in any of the other cases? Is the ratio of surfaces illuminated the same as the ratio of distances?

How far from a candle is the illumination $\frac{1}{16}$ as great as it is 1 ft. from the candle? $\frac{1}{144}$ as great? $\frac{1}{500}$ as great? $\frac{1}{x}$ as great?

Similar Figures. — 1. Compare the surface of a 1-in. cube with the surface of a 2-in. cube, of a 3-in. cube, of a 4-in. cube, and of a 5-in. cube. Compare each with each.

2. Compare the surface of a cube whose edge is x with the surface of a cube whose edge is y.

3. What is the edge of a cube whose surface is x?

4. Compare the surfaces of a 1-in. sphere, a 2-in. sphere, a 3-in. sphere, a 4-in. sphere, and a 5-in. sphere, each with each.

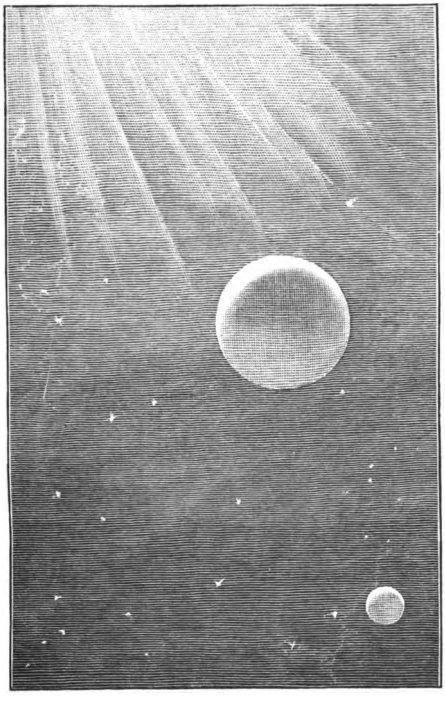
5. What is the ratio of a sphere whose diameter is x to a sphere whose diameter is y? What is the ratio of their surfaces?

6. Compare the lateral surfaces of two equilateral triangular prisms whose edges at the base are 6 in. and 2 in. respectively, and whose altitudes are in the same ratio as the edges of their bases. Are the bases in the same ratio as the lateral surfaces? as the edges of their bases?

7. Two surfaces are similar when their corresponding parts are in the same ratio or when their corresponding lines are in the same ratio. Is the ratio of the corresponding lines equal to the ratio of the corresponding parts?

8. Draw a triangle whose sides are 3, 4, and 5. Draw one similar to it but not equivalent. What is the ratio of their corresponding parts? of their corresponding lines?

9. Similar surfaces are to each other as the squares of corresponding lines. What is the ratio of similar solids?



Distance and Time. — 1. In what direction is the earth turning? Show by gesture.

2. What is it that causes the earth's surface to pass under the sun's rays?

3. How much of the earth's surface is under the sun's rays now?

4. In what time does the now lighted surface pass into the shadow of the earth?

5. To a man in a balloon 5 mi. above Chicago, would

the sun indicate a passage of time if the balloon did not move with the turning earth?

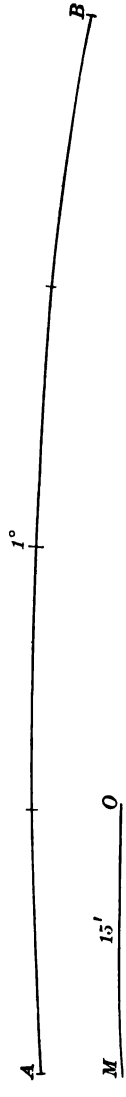
6. If the man were directly over Chicago, on what horizon would he see Chicago disappear?

What time would elapse until it would be directly under him again?

How long a time between its disappearance and reappearance?

In what direction would he look to see it reappear?

If the balloon stopped turning with the earth at six o'clock in the morning, near what horizon would the sun be all the time?



7. What grand divisions and large bodies of water would pass under the man in the balloon before he would see Chicago again?

8. The tops of what mountains would pass near him?

9. Which would pass him first, Erie, N. Y., or Rome, Italy?

10. Which of these cities has sunrise first? Why?

11. How can any particular point in the sky be located?

12. At midnight Chicago is under a particular star; at 6 A.M. it is under a star that is how far from the other? What place or places will then be under the first star?

13. If we wish to locate points in the sky that will be upon our meridian at the even hours, how far apart must we place them?

14. If Chicago is under one of these points at 8 P.M., what time will it be at San Francisco when San Francisco passes under this point?

See p. 243.

1. How many 90 degrees in the circumference of the earth?

2. In what time does the earth turn 90 degrees?

3. How many 15 degrees in 90 degrees? In what time does the earth turn 15 degrees? 30 degrees? 1 degree? 30′? 15′? 60″? 15″?

4. How far does the earth turn in 6 hr.? in 1 hr.? in 4 min.? in 1 min.? in 60 sec.? in 4 sec.?

5. The arc AB is 1 degree in length, or $\frac{1}{360}$ of the circumference of the circle, and it represents the distance that the earth turns in 4 min.

6. Show the part of the arc AB that represents the distance the earth turns in 1 minute. One degree equals how many minutes of distance?

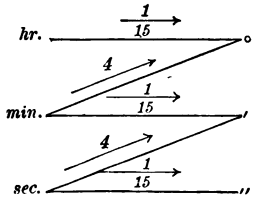
7. The earth turns how many minutes of distance — longitude — in 1 minute of time?

8. In how many seconds of time does the earth turn from M to O? from D to E?

9. In how many seconds does the earth turn 1 min. of distance?

10. In 4 sec. of time the earth turns how many seconds of distance?

11. In any time, what is the ratio of the number of hours to the number of degrees which the earth turns? of the number of minutes of time to the number of degrees? of minutes of time to minutes of distance? of seconds of time to minutes of distance? of seconds of time to seconds of distance?



12. Observe the diagram carefully. Practice mentally picturing it. Drawing the diagram from memory will aid greatly in retaining the mental picture.

13. Give the converse of the above ratios. *Ex.*: 15 is the ratio of the number of degrees to the number of hours.

14. Make sentences like this: If 3 is the number of hours, 45 is the number of degrees.

15. When it is noon in Boston, what is the time on a line extending north and south through Boston?

16. This line is called a meridian, or noonday line.

Show a meridian on the globe. What is a meridian? When it is noon in Boston, is it noon at New York? Explain.

17. When it is noon here, what is the time on the meridian opposite?

NOTE. — *Post meridian* (P.M.) means after noon, and *ante meridian* (A.M.) means before noon.

18. How many meridians are there 1 degree apart?

19. How many meridians 15 degrees apart?

20. How many meridians are represented on the globe?

21. How many degrees apart are they?

22. What is the shape of that part of the earth's surface bounded by two meridians 15 degrees apart?

23. Where is this surface the widest? Where are the degrees the longest? Why?

24. Is the length of a degree in this latitude equal to a degree on the equator? Explain.

The length of a degree on the equator is 60 geographical miles. The length of a degree on the parallel 60 degrees north of the equator is 30 mi.

1. Which of two meridians moves under the sun's rays first? Why?

2. When it is noon here, what is the time 15 degrees west? Why?

How long will it take the meridian 15 degrees west to turn to where we now are?

When the meridian 15 degrees west reaches here, where will we be?

How much earlier is it 15 degrees west than it is here?

3. How many miles west on the equator is it 1 hr. earlier?

4. How long does it take the earth to turn the 900 mi.?

NOTE. — One-sixtieth of a degree is called a minute (′) of longitude.

How long to turn 15 degrees? 30? 45? 20? 25? 16? 10? 5? 1?

5. In what part of an hour does the earth turn 1 degree? In how many minutes?

6. The earth turns each of the different numbers of degrees between 1 and 100 in what time?

7. How far does the earth turn in 4 min.? in 8? in 12? in 6? in 9? in 15? in 30? in 64? in 1 hr.? in 1 hr. 8 min.? in 2 hr. 7 min.?

8. How many minutes of longitude in 1 degree?

9. In 4 min. of time the earth turns how many degrees? How many minutes of distance?

10. When it is noon here, what is the time 15′ east? 15′ west? 45′ east? 45′ west? 75′ east? 75′ west? 85′ west?

11. In how many seconds does the earth turn 15′? In what time does it turn 1′?

Use the following as a basis and question one another:

The earth turns

15 degrees in 1 hr., or 60 min.
1 degree in 4 min.
15′ in 1 min., or 60 sec.
1′ in 4 sec.
15″ in 1 sec.

12. When it is noon here, what is the time 1′ east? 1′ west? 3′ east? 3′ west?

13. How far does the earth turn in 4 sec.? in 8 sec.? in 20 sec.?

14. In what time does the earth turn 37° 16′?

The earth turns 37° in 2 hr. 28 min.
The earth turns 16′ in 1 min. 4 sec.
The earth turns 37° 16′ in 2 hr. 29 min. 4 sec.

The earth turns 30° in how many hours?
The earth turns 7° in how many minutes?

The earth turns 15′ in how many minutes?

The earth turns 1′ in how many seconds?

Then the earth turns 37° 16′ in what time?

Show the corresponding time:

47° 9′; 65° 18′; 54° 47′; 18° 46′.

15. One-sixtieth of 1′ equals 1″ (1 second of longitude). How many seconds of longitude in 1′ of longitude?

16. In 4 sec. the earth turns how many seconds of longitude? The earth turns 15″ of longitude in what time?

17. In 1 sec. the earth turns how many seconds of longitude?

18. In what time does the earth turn 30″? 20″? 35″? 17″? 5″? 10″? 46″?

19. In what time does the earth turn 37° 46′ 20″?

The earth turns 37° in 2 hr. 28 min.

The earth turns 46′ in 3 min. 4 sec.

The earth turns 20″ in $1\frac{1}{3}$ sec.

The earth turns 37° 46′ 20″ in 2 hr. 31 min. $5\frac{1}{3}$ sec.

GREENWICH OBSERVATORY.

20. How far does the earth turn in 2 hr. 31 min. $5\frac{1}{3}$ sec.?

In 2 hr. the earth turns 30°.
In 31 min. the earth turns 7° 45′.
In $5\frac{1}{3}$ sec. the earth turns 1′ 20″.
In 2 hr. 31 min. $5\frac{1}{3}$ sec. the earth turns 37° 46′ 20″.

21. In what direction and how fast would you have to travel on the earth's surface to remain in the same relative position to the sun? Explain.

22. At B it is 7 A.M., and at O 5 A.M. Which is farther east? How many degrees?

23. If your watch keeps Chicago time, will it indicate the right time west of Chicago? Why not? Will it be fast or slow? Are there any directions in which you might travel from Chicago so that your watch would indicate the right time?

24. Locate Greenwich on the globe.

25. What is the greatest distance in degrees that any place can be from Greenwich? Why?

26. Find on the globe the meridian 30° west of Greenwich; another 15° east of Greenwich.

What is the difference in longitude of these two places?

What is their difference in time?

27. When it is noon on the meridian 15° east of Greenwich, what is the time on the meridian 30° west of Greenwich?

28. Make five questions similar to the following: If the difference of time between two places is 2 hr. and 30 min., what is the difference in longitude? *Ans.* 37° 30′.

29. Make five questions similar to the following: If a place is 27° east of Cincinnati, and another 15° west, how many degrees apart are the two places?

What is the difference in time? *Ans.* 2 hr. 48 min.

Table of longitude from the meridian of Greenwich:

Chicago, 87° 37′ 45″ west.	Paris, 2° 20′ east.
Canton, 113° 14′ east.	Philadelphia, 75° 10′ west.
Edinburgh, 3° 11′ west.	Rome, 12° 27′ east.
London, 5′ 48″ west.	San Francisco, 122° 26′ 45″ west.
Montreal, 73° 34′ west.	Washington, 77° 15″ west.
New York, 74° 3′ west.	

Find the difference in the sun time of the following cities:

1. Chicago and New York.
2. Philadelphia and Chicago.
3. Montreal and New York.
4. Paris and Canton.
5. Edinburgh and Canton.
6. Chicago and San Francisco.
7. New York and San Francisco.
8. London and Rome.
9. Rome and New York.
10. San Francisco and London.

What is the difference in longitude corresponding to the following differences in time?

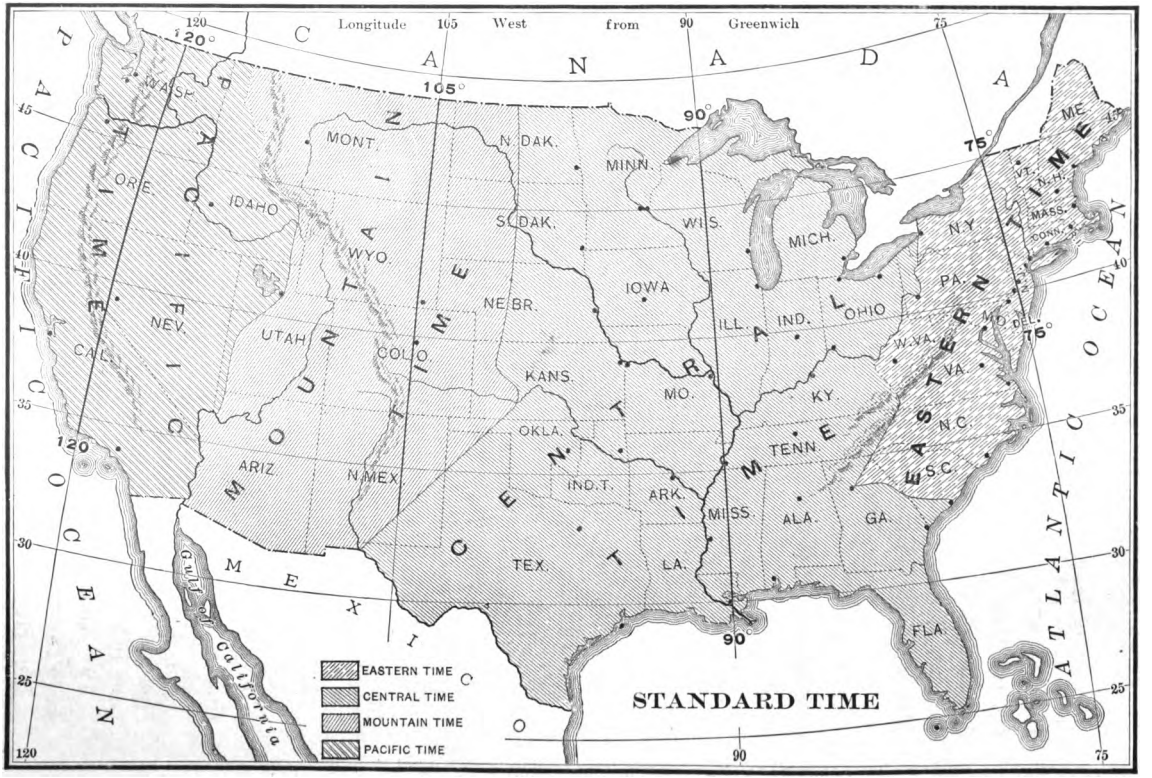
11. 6 hr. 25 min.
12. 2 hr. 19 min.
13. 11 hr. 45 min.
14. 3 hr. 7 min. 11 sec.
15. 2 hr. 40 min. 13 sec.
16. 4 hr. 25 min. 15 sec.

17. When it is 12 o'clock, noon, at Chicago, what is the time at Boston?

18. Do the cities of the United States keep the time of their own meridians? How many meridians of standard time in the United States?

19. About how far west of the 90th meridian do places keep the time of this meridian? how far east?

20. How many times would you reset your watch in


Longitude 105 West from 90 Greenwich
CANADA
PACIFIC OCEAN
ATLANTIC OCEAN
GULF OF MEXICO
Gulf of California
WASH.
ORE.
CAL.
IDAHO
NEV.
MONT.
WYO.
UTAH
COLO.
ARIZ.
N.MEX.
N.DAK.
S.DAK.
NEBR.
KANS.
OKLA.
TEX.
MINN.
IOWA
MO.
ARK.
LA.
WIS.
ILL.
MICH.
IND.
OHIO
KY.
TENN.
MISS.
ALA.
GA.
FLA.
S.C.
N.C.
VA.
W.VA.
PA.
N.Y.
MD.
DEL.
ME.
VT.
N.H.
MASS.
CONN.
IND.T.
MOUNTAIN TIME
PACIFIC TIME
CENTRAL TIME
EASTERN TIME
EASTERN STANDARD TIME
EASTERN TIME
CENTRAL TIME
MOUNTAIN TIME
PACIFIC TIME
EASTERN TIME
CENTRAL TIME
MOUNTAIN TIME
PACIFIC TIME
STANDARD TIME
120°
105°
90°
75°
40
35
30
25

traveling from Portland, Me., to San Francisco? Would you move the hands forward or backward? in returning?

21. Suppose you should travel west around the world keeping standard time, what changes would need to be made in the time of your watch? in traveling east around the world?

22. What is the International Date Line?

When it is 9 A.M. Monday, January 1, at London, is it January 1 all over the world? Where is it not? What date is it there? Tell how this is at 9 P.M. at London; at 6 A.M.; 6 P.M.; 8 A.M.; 8 P.M.

23. When it is noon of Thursday at Chicago, where is it Friday? Is it Wednesday anywhere?

1. I bought a number of boxes of chalk for \$2.50, and sold them for \$3.75, gaining 25¢ on each box. How many boxes did I buy?

2. Two men start from the same place and travel in opposite directions, one at the rate of 3 mi. an hour and the other at the rate of 5 mi. an hour. How far apart are they at the end of 6 hr.?

When they are 96 mi. apart, how many hours have they traveled?

3. If these men had traveled in the same direction, how far apart would they have been in 6 hr.?

4. A silver dollar weighs 412.5 grains, and contains 371.25 grains of pure silver. How much of it is copper?

5. A man having \$$b$ bought 5 calves and had \$$y$ left. $\frac{\$b - \$y}{5}$ = what?

6. A man having \$$m$ bought 3 chairs and had \$$y$ left. What equals the cost of 1 chair?

7. Mary gave to each of her playmates 4¢ and had 20¢ left. If she had given each playmate 9¢, it would have taken all her money. How many playmates had she?

8. A man bought land at $40 an acre and had $500 remaining. Had he paid $60 an acre, it would have taken all his money. How many acres did he buy?

9. The difference between two numbers is 3050; the greater is 6848. What is the less?

10. The number of equal units is 6; one of the equal units is 680. What is their sum?

11. The sum of 6 equal units is 4080. What is one of the equal units?

12. A man earns $25.25 as often as a boy earns $9.50, and in a certain time they together earn $1142.50. How much does each earn?

13. If 4 men earn $8 a day, how many dollars will 48 men earn in 1 day?

14. When oranges are 45¢ a dozen, how much must you pay for 4?

15. If 3 lemons cost 10¢, what do a dozen cost?

16. What is the cost of 2 bu. 1 pk. of apples at 40¢ a pk.?

17. What does it cost to grade 40 rd. of road at $175 a mile?

18. If 18 men can build a wall in 1 da., what part of the wall can 12 men build in the same time?

19. If 12 men can build $\frac{2}{3}$ of a wall in 1 da., in what time can they build the entire wall?

20. If a pole 17 ft. high casts a 3-ft. shadow, what is the length of a shadow cast by a pole 68 ft. high?

21. A pole 150 ft. high fell and broke into two parts; $\frac{2}{3}$ of the longer part was equal to the shorter part. What was the length of each part?

22. When $\frac{8}{9}$ of the time past noon equals $\frac{3}{5}$ of the time to midnight, what is the time past noon?

23. At $\$16\frac{2}{3}$ an acre, how many 6 acres of land can be bought for $\$400$?

24. The number of pupils in a school is 120; the number of boys is 12 more than the number of girls. How many boys in the school?

25. A horse and carriage are worth $\$420$. One-half the value of the carriage equals $\frac{1}{8}$ the value of the horse. Find the value of each.

26. 10 equals $\frac{2}{9}$ of the number of bushels of wheat a man owns which equals $\frac{2}{7}$ of the number of bushels of oats he owns. How many bushels of each has he?

27. To what scale is a map drawn which represents 36,000 sq. mi. and is 10 in. long and 8 in. wide?

28. What is the cost of fencing a 4-by-9 rectangular field of $3\frac{3}{5}$ acres at $\$1.75$ a rod?

29. A rectangular cistern is 12 ft. long, 8 ft. wide, and 11 ft. deep; the water in it is 5 ft. deep. If a rectangular stone 3 ft. $\times$ 2 ft. $\times$ 1 ft. be dropped into the cistern, how much will the water rise?

30. If $\frac{8}{9}$ of a block on State Street is worth $\$x$, what equals the value of $\frac{1}{27}$ of a block?

Express in one sentence the answer to each of the next three questions:

31. If $\frac{4}{17}$ of a farm is worth $\$577$, what equals the value of $\frac{2}{3}$ of the farm? What is the value?

32. If $\frac{16}{18}$ of a lot is worth $\$850$, what equals the value of the lot? What is the value?

33. If $\frac{2}{3}$ of a lb. of coffee costs $26¢$, what does $16\frac{3}{4}$ lb. cost?

34. If A can do $\frac{1}{4}$ and B $\frac{1}{8}$ of a piece of work in x da., what equals the time in which they can do the work together?

35. If 5 men mow 27½ A. of grass in a certain time, what equals the number of men that can mow x A. in the same time?

36. If 3 men can mow 20 A. of grass in 8 da., how many men can mow 10 A. in 6 da.? Show this statement to be true:

$\frac{10 \cdot 4 \cdot 3}{20 \cdot 3}$ = number of men required to mow 10 A. in 6 da.

37. What is the value of a half section of land at \$16½ an acre?

38. If the expenses of a family of 12 persons amount to \$80 in 10 wk., how long will \$400 support a family of 9 persons?

39. If \$200 yields \$64 in 4 yr., at the same rate how long does it take \$500 to yield \$160?

What equals the number of dollars \$500 would yield in 4 years?

Then \$500 would yield \$64 in what time?

40. What will a board 20 ft. long and 9 in. wide cost at \$30 a thousand?

41. A girl sold flowers at 20¢ and gained a sum equal to ⅓ of the cost. What was the cost?

42. If John can build a fence in ¼ of a month, and James in ⅓ of a month, in what time can they together build it?

43. A woman bought 246 apples at the rate of 3 for 4¢, and sold them at the rate of 2 for 3¢. What did she gain?

44. What is the surface of a brick 2 in. × 4 in. × 8 in.?

45. What is the distance around a room which requires to cover its floor 5 strips of carpet each 5 yd. long and ¾ of a yd. wide?

46. How many half-inch cubes in a 2-in. cube?

47. If a block of wood 1 yd. in each dimension is sawed

vertically through the middle from front to back, and vertically through the middle from right to left, and horizontally through the middle, what are the dimensions of one of the parts?

48. How many 16 hundred-thousandths in 96 thousandths?

49. Change $\frac{4}{700}$ to a decimal.

50. Express as a common fraction .00375.

51. What decimal of a ream is 3.8 quires?

52. What decimal of an acre is 8.6 sq. rd.?

53. How much water must be added to 750 gal. of wine worth $5 a gal. that the mixture may be worth $4 a gal.?

54. A piece of paper is 7 in. × 11 in. Cut it into 2 pieces so that there are 11 sq. in. more in one piece than in the other.

55. Make and solve 5 problems similar to the 54th.

56. $6820 were divided between two men. The difference between their shares was $512. What did each receive?

57. There were 51,000 votes cast at the city election. Of these one candidate received 1550 more than the other. How many votes did each receive?

58. a is the perimeter of an equilateral triangle and b its altitude. What equals its area?

59. If the base of a triangle is 75 rd. and the altitude 40 rd., how many acres does it contain?

60. At $375 an acre, what is the value of a triangular field whose base is 60 rd. and altitude $42\frac{2}{3}$ rd.?

61. At $.20 a foot, what is the cost of fencing a triangular field the three sides of which are in the ratio of 5, 6, and 8, the shortest side being 120 ft.?

62. The gable ends of a barn are each 20 ft. wide, and the perpendicular height of the ridge above the eaves is 8 ft. How many feet of boards will be required to board both gables?

63. What is the altitude of a triangle having an area of 100 sq. ft. and a base of $12\frac{1}{2}$ ft.?

64. Paid \$16,000 for a triangular farm at \$800 an acre. If the base of the field is a mile, what is its altitude?

65. How many square feet are there on the sides and bottom of a box without cover 2 ft. × 3 ft. × 4 ft.?

66. If the oil in a tank will run through an opening 3 in. × 3 in. in 20 min., in what time will it run through an opening 2 in. × 2 in.?

67. At \$5 per cord, what is a pile of wood worth that is 60 ft. long, 4 ft. wide, and 8 ft. high?

68. A grocer exchanges flour worth \$6 a barrel for wood worth \$8 a cord. If the owner of the wood asks \$9.50 a cord, to increase in the same proportion what price should the grocer ask for his flour?

69. Mr. Jones invested \$1250 and gained an amount equal to 10% of this sum every year for 3 yr. What was his profit?

70. What is the tax of 3% on property valued at \$700?

71. A lawyer received 3% for collecting a debt of \$350. What was his commission?

72. What amount of money will pay a note of \$200 due in $2\frac{1}{2}$ yr. at 8%?

73. A paid \$128 for goods, which was $33\frac{1}{3}$% less than their value. What was their value?

74. If to 30 qt. of wine 1 gal. of water is added, what % of the mixture is water?

75. When selling hay, what % does a farmer gain if the scales on which his hay is weighed mark 1 cwt. at 94 lb.? When paid for 18 tons of hay at \$5 a ton, how much does he gain?

76. If by selling goods at $16\frac{2}{3}$% profit a merchant clears \$42, what was the cost of the goods?

77. In a mixture of butter and tallow the tallow equals

$12\frac{1}{2}\%$ of the butter. The tallow equals what % of the mixture?

78. By selling a house for \$800 a man lost 10%. What should he have sold it for to gain 10%?

79. A boy paid \$1.50 for 60 newspapers. Allowing that $\frac{1}{6}$ will be unsold, for how much apiece must he sell the others to gain $33\frac{1}{3}\%$?

80. A 2-ft. square equals how many 4ths of a square foot?

81. 27 equals how many 4ths of 12?

82. 9 ft. equals how many 8ths of 12 ft.?

83. 4 ft. equals how many 7ths of 3 ft.?

84. One of the faces of a $\frac{1}{2}$-in. cube equals what % of one of the faces of a 1-in. cube?

85. E bought a basket of grapes for $\frac{4}{5}$ of the market price. The saving equaled what part of the cost?

86. If \$3 is lost by selling 5 yd. of velvet at a loss of 30%, what was the cost of the velvet per yd.?

87. A milliner bought \$150 worth of goods and sold them for \$120. How many cents did she lose on each dollar invested?

88. If a merchant sells $\frac{3}{5}$ of his stock for a sum equal to the cost of $\frac{4}{5}$ of it, what % is gained?

89. What is the thickness of a block of stone which weighs 1650 lb. and is 3 ft. long and 2 ft. wide, if a cubic foot of stone weighs 150 lb.?

90. A room 30 ft. long requires 80 yd. of carpet $\frac{3}{4}$ of a yd. wide. How wide is the room?

91. A brick is 8 in. × 4 in. × 2 in. How many will it take to lay $\frac{1}{4}$ mi. of 8-ft. sidewalk, the brick being laid on one of the largest surfaces?

92. I send my agent \$420 to invest in plows at \$10 each after deducting his commission at 5%. How many plows can be purchased?

93. What is the tax collected on $6000 worth of property at $\frac{1}{8}\%$?

94. When the rate of taxation is $\frac{1}{4}\%$, what amount of property will give a tax of $18.75?

95. At what rate must property valued at $100,000 be taxed to raise $500?

96. How many board feet of lumber are required to make from 2-in. lumber a cubical box that will hold 1 cu. ft. water?

97. How many pickets will be required to fence a garden 8 ft. × 12 ft. if the pickets are 4 in. wide and are placed 2 in. apart?

98. If a horse eats 2 bu. of oats in x da., in how many x da. will 2 horses eat 26 bu.?

99. If 4 men plow x A. in $8\frac{1}{2}$ hr., how many x A. can 8 men plow in 20 hr.?

100. If $\frac{2}{7}$ of a yd. of ribbon costs $\$\frac{3}{8}$, what does $\frac{1}{2}$ yd. cost?

101. Show by diagram the answer to the following: $\frac{2}{3}$ of a unit increased by $\frac{1}{4}$ of the unit and this increased by the unit 34 will give twice the unit. What is it?

102. A man bought peaches at $1 a pk., and half as many pecks of apples at half as much a peck. The cost of both was $12.50. How many pecks of each did he buy?

103. A merchant bought a number of yards of silk at $3 a yd., and $\frac{1}{3}$ as many yards of cashmere at $\frac{1}{3}$ as much a yd. The cost of the whole was $175. How many yards of each did he buy?

104. A man sold $\frac{3}{7}$ of his corn crop and had 720 bu. left. How much had he at first?

105. The sum of two numbers is 17935, and their difference equals $\frac{2}{3}$ of the greater number. What are the numbers?

106. $\frac{3}{4}$ of a bbl. of apples at \$7 a bbl. will pay for what part of a ton of coal at \$9 a ton?

107. If goods are bought at $\frac{3}{8}$ off the retail price, what % is gained by selling at the retail price?

108. An agent collected 50% of a debt of \$1200 and charged $3\frac{1}{2}$% commission. What amount will he remit?

109. By buying pears at the rate of 3 for 6¢, $\frac{1}{2}$ is saved. What is the retail price per pear?

110. At what price must cloth costing \$1.20 a yd. be marked so that 10% may be deducted from the marked price and it be sold at a profit of 20%?

111. Find the difference between $37\frac{1}{2}$% of \$2400 and $.37\frac{1}{2}$% of the same.

112. What % of an A. is 30 sq. rd.? 5 sq. ft.?

113. $\frac{1}{2}$ a mi. is what % of 40 rd.?

114. A merchant's assets are \$27,000 and his liabilities \$87,000. What % can he pay?

115. The premium for insuring a stock of goods at $1\frac{1}{2}$% was \$250. What was the amount invested?

116. If 60 apples are worth \$1 in currency, what in currency are 100 apples worth?

117. When a paper dollar is worth 60¢ in gold, what is the price of gold?

118. Property was bought for \$2759 and sold for \$2516. What was the % of loss?

119. Sold land at 120% of its cost and thereby gained \$7 a ft. What was the cost per ft.?

120. By selling a house for \$1200 I shall lose $12\frac{1}{2}$%. For how much should I sell it to gain $12\frac{1}{2}$%?

121. A fruit dealer lost $33\frac{1}{3}$% of a lot of apples and sold the remainder at a gain of 50%. Required the % of gain or loss.

122. A farmer lost $16\frac{2}{3}$% of his stock and sold the remainder at a profit of 50%. What was the % of gain or loss?

123. A boy bought a knife for 60¢ and sold it for 20% more than it cost and 25% less than he asked for it. How much did he ask?

124. A farmer sold a cow to a shipper at a gain of 20%; the shipper sold to a butcher at a gain of $33\frac{1}{3}$%; the butcher realizing $60 for the cow made 10%. What was the price the farmer received for the cow?

125. What is the entire surface of a block of wood $1\frac{1}{2}$ ft. × $5\frac{1}{2}$ ft. × 3 ft.?

126. How many cords of wood can be piled against the wall of a room 16 ft. × 16 ft. × 8 ft., so that one end of each stick will touch the wall?

127. If berries are bought at the rate of 12 qt. for $1 and sold at 8 qt. for $1, what is the % of profit?

128. A man invests his money in wheat, which sells for $2640 more than he paid for it. His money then equals $133\frac{1}{3}$% of what he had at first. How much did he have at first?

129. By selling wine at $1.80 a pt. a dealer gains 40%. What would be the selling price at a loss of 40%?

130. A merchant sold lead pencils for 3¢ each, which was at a gain of 25%. What did the pencils cost him?

131. At what rate % simple interest will $100 double itself in 10 yr.?

132. If by buying a 5-lb. box of paper for $1, $\frac{1}{5}$ is saved, what is the cost when bought by the single pound?

133. A strip of land containing 2 A. is $\frac{1}{4}$ mi. long. How wide is it?

134. A and B can mow a field in 4 da., and A can do only $\frac{2}{3}$ as much work as B. How long will it take each alone to mow the field?

135. If a creditor agrees to accept 60¢ on the dollar and then discounts 3% for immediate payment, what does he lose on a claim of $28,000?

136. At what discount must stock paying a half-yearly dividend of $3\frac{1}{2}\%$ be purchased to enable the buyer to realize 8% on his investment?

137. How high must a box be to hold 80 2-in. cubes if its other dimensions are 8 in.? Why?

138. What is the area of the entire surface of a 4-ft. cube?

139. How wide must a paper border of a square yard be to contain 432 sq. in.?

140. How many paper boxes 3 in. long, 2 in. wide, and 2 in. deep can be packed in a cu. ft. of space?

141. A sidewalk a mile long contains $\frac{3}{4}$ of an acre. How wide is it?

142. If your schoolroom is 16 ft. high and contains 12,000 cu. ft., how many sq. ft. are there in the floor?

143. If a cu. ft. of water weighs 1000 oz. and expands $\frac{1}{10}$ in. in freezing, how many tons of ice are there on a 4-A. pond when the pond is frozen 8 in. deep?

144. What is the cost of painting a blackboard 20 ft. long and $3\frac{1}{2}$ ft. wide at 36¢ a sq. yd.?

145. How many cu. ft. in $5\frac{1}{2}$ cd. of wood?

146. A reaper cuts a 6-ft. swath around a field that is 48 rd. long and 540 ft. wide. How many sq. ft. in the swath?

147. How high do you ascend in going up 3 flights of stairs, the first of 15 steps, and the other two 14 steps each, if each step is $9\frac{1}{2}$ in. high?

148. What is the cost of 5 boards, each $\frac{3}{4}$ in. thick, 16 ft. long, and 18 in. wide, at $30 per M.?

149. How many rods of fence will be required to enclose a rectangular field containing 20 A., one side of which is 128 rd.?

150. A reservoir is 36 ft. long and 25 ft. 3 in. wide. How many cu. ft. of water must be drawn off to lower the surface 4 in.?

151. How many posts 6 ft. apart will enclose a section of land?

152. If a map is drawn to the scale of 125 mi. to an in., what is the area of a state the dimensions of which on the map are $1\frac{3}{8}$ in. by 2 in.?

153. If 1000 shingles laid 4 in. to the weather will cover 100 sq. ft. of surface, how many shingles so laid will be required for the roof of a barn each side of which is 40 ft. wide and 120 ft. long?

154. A rectangle $3\frac{1}{7}$ in. long and $\frac{1}{4}$ in. wide equals what part of a 2-in. square?

155. How many feet of lumber in a stick of timber 30 ft. long and 25 in. square?

156. John and William can saw a pile of wood in 14 da., and John can saw only $\frac{3}{4}$ as much as William. In what time can each alone saw the wood?

157. If x is the number of men required to do a piece of work in a given time, how many men will be required to do 4 times as much work in $\frac{1}{3}$ of the time?

158. If 8¢ buys a 6-oz. loaf when flour is \$10 a bbl., how large a loaf does it buy when flour is \$7 a bbl.?

159. If $\frac{1}{10}$ is saved by buying a dozen handkerchiefs for \$4.50, what is the price of a handkerchief if bought singly?

160. If 7 yd. of goods are required for a suit and the cloth shrinks 10% in sponging, how many yards must I buy for a suit?

161. If 13 yd. of gingham are required to make a dress, which shrinks $\frac{1}{12}$ in washing, how many yards must be bought to make a dress, allowing for shrinkage?

162. If canvas shrinks 10%, how many yards should be purchased so that after shrinking it may cover a platform 27 ft. long and 26 ft. wide? a platform 7 yd. long and $5\frac{1}{2}$ yd. wide?

163. If you buy wool at 24¢ a lb. and it loses $\frac{1}{7}$ of its weight in cleansing, for how much must it be sold to make 10¢ per lb.?

What remains of a lb. after cleansing?

$\frac{7(24+10)}{6} = ?$

For what, then, must $\frac{6}{7}$ of a lb. be sold that there may be no loss?

For what, then, must $\frac{6}{7}$ of a lb. be sold that 10¢ may be gained? For what, then, must a lb. be sold?

164. The sum of A and B equals x. $\frac{2}{3}$ of A equals $\frac{3}{5}$ of B. A equals what part of x?

What is the relation of A to $\frac{2}{3}$ of A?

What, then, is the relation of A to $\frac{3}{5}$ of B?

$\frac{3}{5}$ of $\frac{3}{5}$ equals what?

What, then, is the relation of A to x?

165. The sum of C and D equals Y. $\frac{2}{3}$ of C equals $\frac{2}{4}$ of D. What is the ratio of C to Y?

166. The time past noon and the time to midnight equals what? $\frac{2}{3}$ of the time past noon equals $\frac{3}{5}$ of the time to midnight. What is the time?

What is the relation of the time past noon to $\frac{2}{3}$ of the time past noon?

What, then, is the relation of the time past noon to $\frac{3}{5}$ of the time to midnight?

$\frac{9 \cdot 12}{19} = ?$

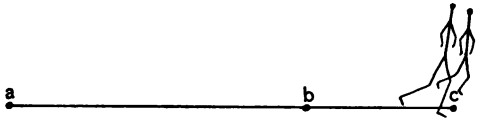
$\frac{3}{5}$ of $\frac{3}{5}$ equals what?

What, then, is the relation of the time past noon to the time to midnight?

What, then, is the relation of the time past noon to the time from noon until midnight?

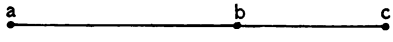
167. B is x mi. in advance of A. A travels 3 times as fast as B. What equals the distance A travels before overtaking B?

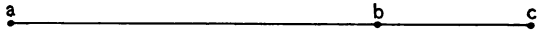
The distance B travels in any time equals what part of the distance A travels in the same time?



Then the distance B is in advance of A equals $\frac{3 \cdot x}{2} = ?$ what part of the distance A travels? Why?

What is the relation of the distance that A travels to the distance B is in advance of A at the time of starting?





168. What ratios do you find in each of these diagrams? Make problems of the nature of those above and involving these ratios.

169. A car travels 8 times as fast as a tramp. The tramp is x mi. in advance of the car. What equals the distance the car must travel before overtaking the tramp?

In any time, the distance the tramp travels equals what part of the distance traveled by the car?

The distance the tramp is in advance of the car $\frac{8 \cdot x}{7} = ?$ equals what part of the distance the car travels?

What is the ratio of the distance the car travels to the distance the tramp is in advance of the car at the time of starting?

170. The minute-hand moves how many times as fast as the hour-hand? In any time the distance moved by the hour-hand equals what part of the distance moved by the minute-hand? The hour-hand is x minutes in advance of the minute-hand. What equals the time in which the

minute-hand will overtake the hour-hand? What is the ratio of the distance the hour-hand is in advance of the minute-hand to the distance the minute-hand moves before overtaking the hour-hand?

What is the ratio of the distance moved by the minute-hand to the distance the hour-hand is in advance of the minute-hand at the time of starting?

$\frac{12 \cdot x}{11} = ?$

171. What is the time of day when $\frac{2}{3}$ of the time past midnight equals $\frac{5}{6}$ of the time to noon?

172. What is the hour of day when $\frac{2}{5}$ of the time past noon equals $\frac{3}{7}$ of the time to midnight?

173. If the time past noon increased by 40 min. equals $\frac{5}{8}$ of the time from noon to midnight, what time is it?

174. At what time between 5 and 6 o'clock are the hour and minute hands of a watch together? At what time between 3 and 4 o'clock?

175. A runs 12 rd. while B runs 11. In a race of 6 min., how much must B be given the start that they may reach the goal together?

176. John is 30 rd. in advance of Frank, but Frank runs 8 yd. while John runs 5. How far must Frank run to overtake John?

177. If a steamer sails 8 mi. an hr. down stream and 6 mi. an hr. up stream, how far can it go down and back in 12 hr.?

178. Alice starts to school, and after she has traveled $\frac{1}{2}$ of the distance George starts from the same place to overtake her, traveling three times as fast. What part of the distance from the starting point to the schoolhouse does George travel before overtaking Alice?

179. Make and solve 3 problems similar to the 178th.

180. How many miles does a boy walk in plowing a 12-A. square field and turning a 10-in. furrow?

181. Stock bought at 12% premium pays 5% on the investment. What would it pay if bought at 15% discount?

182. When stock is bought at a discount of 20% and sold at a discount of 8%, what % is gained?

183. What was the face of a draft bought for $2730, when exchange was at $\frac{1}{2}$% discount?

184. If stock bought at 10% discount pays 8% on the investment, at what price should the same stock be bought to pay 10%?

185. If gold is at $16\frac{2}{3}$% premium, what is the corresponding discount on currency?

186. If a man asks 40% profit, but falls 10% on his asking price, what is the rate of profit?

187. A man sold a cow for $45 and thereby gained 25%. If he had sold it for $30, would he have gained or lost, and what %?

188. A grocer invested $215 as follows:

He spent 60% of the money for pears at $1.50 a bu.; 30% for apples at $87\frac{1}{2}$¢ a bu., and the remainder for potatoes at $.40 a bu. How many bu. of each did he buy?

189. Two horses were sold for $189 each. On one there was a gain of 10% and on the other a loss of 10%. Find the gain or loss on the sale of both.

190. A man has two pictures and a frame worth $20. If he puts the frame on the first picture, it will be worth $\frac{2}{3}$ as much as the second picture; but if he puts the frame on the second picture, it will be worth $2\frac{3}{4}$ times as much as the first picture. What is the value of the pictures?

191. My semi-annual income from a stock investment that yields 10% is $106. How many shares do I own?

192. A lawyer earns $450 collecting money at 5%. What amount does he collect?

193. If goods are bought at $\frac{1}{4}$ of their value and sold for 25% more than their value, what is the gain %?

194. A florist buys flowers at $\frac{1}{3}$ off from the retail price. What % does he make by selling at the retail price?

195. If wool that costs \$.60 per lb. shrinks 5% in cleansing, at what price per lb. must it be sold to gain $33\frac{1}{3}$% on the cost?

196. I buy 8% stock at 110. What % do I receive on my investment?

197. A merchant received for a lot of goods \$6520. He had deducted 5% from the face of the bill and still found that he had made 12%. What did he pay for the goods?

198. If \$80 yields \$6.30 in a given time, what principal will yield \$104.50 in the same time?

199. If \$475 yields \$$x$ in a certain time, what will \$187.50 yield in the same time?

200. A road $\frac{3}{4}$ mi. long and 5 rd. wide is run through a man's farm. He is paid \$30 an acre for his land. How much does he receive?

201. $\frac{2}{3}$ of Mr. Wilson's property equals $\frac{3}{4}$ of Mr. Brown's. The difference in the value of the two men's property is \$724. What is the value of each man's property?

202. The fore wheels of a carriage are each 9 ft. in circumference, and the hind wheels are each $1\frac{1}{2}$ times as many ft. in circumference. If each fore wheel revolve 6300 times in going a certain distance, how many times will each hind wheel revolve?